12-MTL-A2, TRANSPORT PHENOMENA IN MATERIALS ENGINEERING

NATIONAL EXAMINATIONS DECEMBER 2019

3 hours duration

NOTES

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. The examination is an **open book exam.** One textbook of your choice with notations listed on the margins etc., but no loose notes are permitted into the exam.

3. Candidates may use any **non-communicating** calculator.

4. Regular graph papers will be provided.

5. All problems are worth **25 points**.

6. Any **four questions** constitute a complete paper.

7. Only the **first four** questions as they appear in the answer book will be marked.

8. State all assumptions clearly.

1. A polymer melt follows the power law for variation of shear stress (τ_{yx}) with shear strain rate (dV_x/dY) given by the following equation:

$$\tau_{yx} = -\eta_o [dV_x/dY]^n$$

where η_o → zero-shear viscosity
V_x → local velocity in the x-direction
n → Power law index

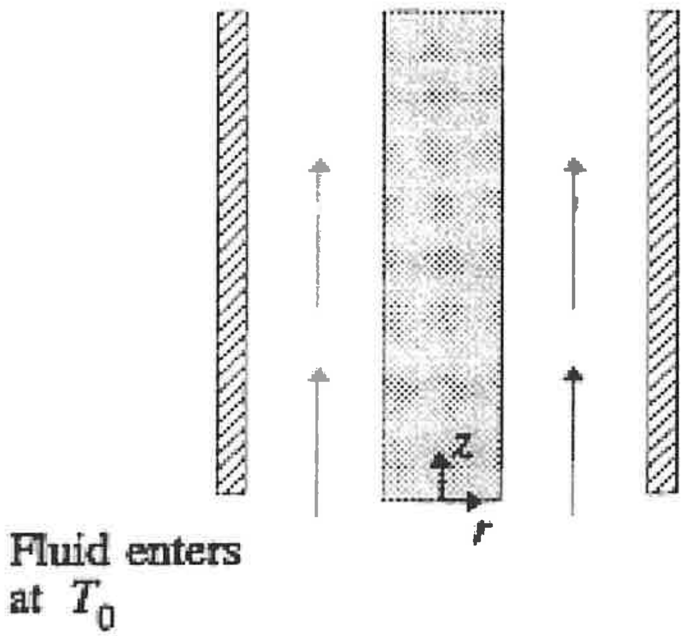
Derive an equation for the velocity profile (V_x) and volumetric flow rate (Q) for flow between two parallel plates. The direction between the parallel plates is the Y-direction.

2. A 2.5 mm thick, 2.5 m long square steel plate is removed from an oven at 430 K to an atmosphere at 295 K. Calculate the initial heat loss (W) for the following conditions:

a) **[15 points]** The plate is hung horizontally.

b) **[10 points]** The plate is hung vertically.

3. A liquid of constant density and viscosity flows upwards in the annulus between very long and concentric cylinders ($R_2 \geq r \geq R_1$) as shown below:



The inner cylinder is electrically heated and supplies a constant uniform heat flux (q_1) to the liquid. The outer cylinder is maintained at a constant temperature T_0. Assume that both flow and temperature are fully developed.

a) [15 points] Obtain an equation velocity V_z.

b) [7 points] Write the equation of energy and state your assumptions.

c) [3 points] Write appropriate boundary conditions for energy equation.

4. A flat ceramic mold of thickness L is used for solidification of metals. The outside surface of the mold loses heat to the surroundings with a constant heat transfer coefficient of 150 $W/m^2.K$. The temperature in the mold is at steady state except at very early times.

a) **[8 points]** Derive an equation for solidification thickness as a function of time.

b) **[3 points]** A plate of 38-mm thick nickel is cast in a 10-mm thick ceramic shell mold. Calculate the solidification time.

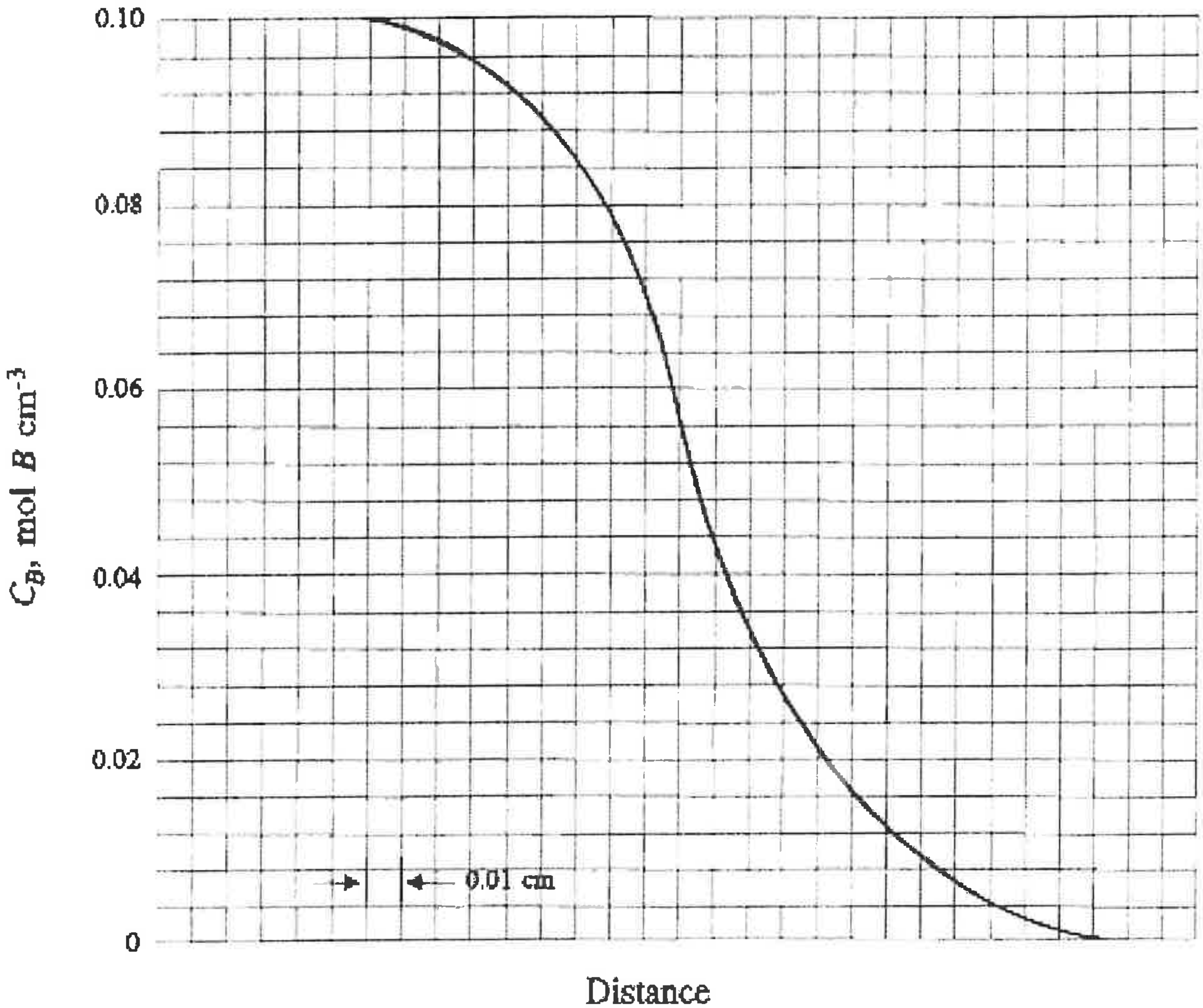
c) **[9 points]** The flat ceramic mold is replaced with a cylindrical shell of thickness L. Derive an equation for solidification thickness as a function of time.

d) **[5 points]** A cylinder of 38-mm diameter nickel is cast in a 10-mm thick ceramic shell mold. Calculate the solidification time.

DATA:

Thermal conductivity of ceramic shell = 0.7 W/m.K
Effective latent heat of fusion of nickel = 291 kJ/kg
Effective density of nickel = 7.85 g/cm^3

5. An alloy composed of metals A and B has a face centered cubic structure (FCC) at a temperature of 1200 °C. They are allowed interdiffuse for about 28 hours and the following concentration profile is obtained:



Determine the value of the interdiffusion coefficient (D_{AB}) for $C_B = 0.02$ mol/cm^3.

APPENDIX A

Summary of the Conservation Equations

Table A.1 The Continuity Equation

$$\frac{\partial \rho}{\partial t}+(\nabla \cdot \rho \vec{u})=0$$	(1.1)
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u_x)+\frac{\partial}{\partial y}(\rho u_y)+\frac{\partial}{\partial z}(\rho u_z)=0$$	(1.1a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta)+\frac{\partial}{\partial z}(\rho u_z)=0$$	(1.1b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 u_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\rho u_\theta \sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho u_\phi)=0$$	(1.1c)

Table A.2 The Navier-Stokes equations for Newtonian fluids of constant ρ and μ

	$$\frac{\partial \vec{u}}{\partial t}+(\vec{u}\cdot\nabla)\vec{u}=-\frac{1}{\rho}\nabla P+\vec{g}+\nu(\nabla^2\vec{u})$$	(A2)
Rectangular coordinates (*x*, *y*, *z*)		
x-component	$$\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+g_x+\nu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)$$	(A2a)
y-component	$$\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+g_y+\nu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)$$	(A2b)
z-component	$$\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)$$	(A2c)

Cylindrical coordinates (r, θ, z)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_r)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right] \tag{A2d}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}+\frac{u_r u_\theta}{r} = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta+\nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ru_\theta)}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right] \tag{A2e}$$

z-component

$$\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z}+g_z+\nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right] \tag{A2f}$$

Spherical coordinates (r, θ, ϕ)

r-component

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_r}{\partial \phi}-\frac{u_\theta^2}{r}-\frac{u_\phi^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r}+g_r +\nu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2u_r\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_r}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial \phi^2}\right] \tag{A2g}$$

θ-component

$$\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\left(\frac{u_\phi}{r\sin\theta}\right)\frac{\partial u_\theta}{\partial \phi}+\frac{u_r u_\theta}{r}-\frac{u_\phi^2}{r}\cot\theta = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+g_\theta +\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\theta\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial \phi^2}\\ +\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\phi}{\partial \phi}\end{array}\right] \tag{A2h}$$

ϕ-component

$$\frac{\partial u_\phi}{\partial t}+u_r\frac{\partial u_\phi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\phi}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_r u_\phi}{r}+\frac{u_\theta u_\phi}{r}\cot\theta = -\frac{1}{\rho r\sin\theta}\frac{\partial P}{\partial \phi} +g_\varphi+\nu\left[\begin{array}{l}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\phi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(u_\phi\sin\theta\right)\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\phi}{\partial \phi^2}\\ +\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial \phi}+\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial u_\theta}{\partial \phi}\end{array}\right] \tag{A2i}$$

Table A.3 The Energy Equation for Incompressible Media

$$\rho c_P\left[\frac{\partial T}{\partial t}+(\vec{u}\cdot\nabla)(T)\right]=[\nabla\cdot k\nabla T]+\dot{T}_G \tag{A3}$$

Rectangular coordinates (*x*, *y*, *z*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G \tag{A3a}$$

Cylindrical coordinates (*r*, *θ*, *z*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial\theta}+u_z\frac{\partial T}{\partial z}\right]=\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(k\frac{\partial T}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{T}_G \tag{A3b}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

$$\rho c_P\left[\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial\theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial T}{\partial\phi}\right]=$$
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2k\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(k\sin\theta\frac{\partial T}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\phi}\left(k\frac{\partial T}{\partial\phi}\right)+\dot{T}_G \tag{A3c}$$

Table A4: The continuity equation for species *A* in terms of the molar flux

$$\frac{\partial C_A}{\partial t}=-\left(\nabla\cdot\vec{N}_A\right)+\dot{R}_{A,G} \tag{4.}$$

Rectangular coordinates (*x*, *y*, *z*)

$$\frac{\partial C_A}{\partial t}=-\left(\frac{\partial[N_A]_z}{\partial x}+\frac{\partial[N_A]_y}{\partial y}+\frac{\partial[N_A]_z}{\partial z}\right)+\dot{R}_{A,G} \tag{4a}$$

Cylindrical coordinates (*r*, *θ*, *z*)

$$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r}\frac{\partial}{\partial r}[rN_A]_r+\frac{1}{r}\frac{\partial}{\partial\theta}[N_A]_\theta+\frac{\partial}{\partial z}[N_A]_z\right\}+\dot{R}_{A,G} \tag{4b}$$

Spherical coordinates (*r*, *θ*, *ϕ*)

$$\frac{\partial C_A}{\partial t}=-\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2[N_A]_r\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left([N_A]_\theta\sin\theta\right)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}[N_A]_\phi\right\}+\dot{R}_{A,G} \tag{4c}$$

Table A.5: The continuity equation for species *A*

$$\frac{\partial C_A}{\partial t}+(\vec{u}\cdot\nabla)C_A=D_A\nabla^2 C_A+\dot{R}_{A,G}$$	(5)
Rectangular coordinates (*x*, *y*, *z*) $$\frac{\partial C_A}{\partial t}+u_x\frac{\partial C_A}{\partial x}+u_y\frac{\partial C_A}{\partial y}+u_z\frac{\partial C_A}{\partial z}=\frac{\partial}{\partial x}\left(D\frac{\partial C_A}{\partial x}\right)+\frac{\partial}{\partial y}\left(D\frac{\partial C_A}{\partial y}\right)+\frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right)+\dot{R}_{A,G}$$	(5a)
Cylindrical coordinates (*r*, *θ*, *z*) $$\frac{\partial C_A}{\partial t}+u_r\frac{\partial C_A}{\partial r}+\frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta}+u_z\frac{\partial C_A}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C_A}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(D\frac{\partial C_A}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(D\frac{\partial C_A}{\partial z}\right)+\dot{R}_{A,G}$$	(5b)
Spherical coordinates (*r*, *θ*, *ϕ*) $$\frac{\partial C_A}{\partial t}+u_r\frac{\partial C_A}{\partial r}+\frac{u_\theta}{r}\frac{\partial C_A}{\partial \theta}+\frac{u_\phi}{r\sin\theta}\frac{\partial C_A}{\partial \phi}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 D\frac{\partial C_A}{\partial r}\right)$$ $$+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(D\sin\theta\frac{\partial C_A}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(D\frac{\partial C_A}{\partial \phi}\right)+\dot{R}_{A,G}$$	(5c)

Table 1: Properties of Dry Air at One Atmosphere

Density and specific heat are from Hilsenrath (1955); thermal conductivity and viscosity are from Touloukian (1970); other values are calculated.

T $[K]$	ρ $\left[\frac{kg}{m^3}\right]$	μ $\left[10^{-6}\frac{N\cdot s}{m^2}\right]$	κ $\left[10^{-3}\frac{W}{m\cdot K}\right]$	C_p $\left[\frac{J}{kg\cdot K}\right]$	ρ/μ $\left[10^3\frac{s}{m^2}\right]$	$g\beta/(\nu\alpha)$ $\left[10^6\frac{1}{m^3\cdot K}\right]$	α $\left[10^{-6}\frac{m^2}{s}\right]$
200	1.7690	13.36	18.10	1006.4	132.4	638.6	10.17
210	1.6842	13.92	18.95	1006.1	121.0	505.2	11.18
220	1.6071	14.47	19.80	1005.7	111.1	404.2	12.25
230	1.5368	15.01	20.63	1005.6	102.4	327.0	13.35
240	1.4728	15.54	21.45	1005.5	94.8	267.3	14.49
250	1.4133	16.06	22.26	1005.4	88.0	220.4	15.67
260	1.3587	16.57	23.05	1005.5	82.0	183.3	16.87
270	1.3082	17.07	23.84	1005.5	76.6	153.6	18.12
280	1.2614	17.57	24.61	1005.7	71.8	129.6	19.40
290	1.2177	18.05	25.38	1006.0	67.5	110.1	20.72
300	1.1769	18.53	26.14	1006.3	63.5	94.1	22.07
310	1.1389	19.00	26.87	1006.8	59.9	80.9	23.43
320	1.1032	19.46	27.59	1007.3	56.7	70.0	24.83
330	1.0697	19.92	28.30	1007.9	53.7	60.8	26.25
340	1.0382	20.37	29.00	1008.5	51.0	53.1	27.70
350	1.0086	20.81	29.70	1009.2	48.5	46.5	29.18
360	0.9805	21.25	30.39	1010.0	46.1	41.0	30.69
370	0.9539	21.68	31.07	1010.9	44.0	36.2	32.22
380	0.9288	22.11	31.73	1012.0	42.0	32.1	33.76
390	0.9050	22.52	32.39	1013.0	40.2	28.6	35.33
400	0.8822	22.94	33.05	1014.2	38.5	25.5	36.94