National Exams - December, 2016

98-Civ-B9

Applications of the Finite Element Method

3 hours duration

Notes:

1. There are five (5) pages in this examination, including the front page.
2. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. This is an open book examination.
4. Candidates may use any non-communicating calculator.
5. **Attempt to <u>answer only two</u> from the three proposed problems**
6. All problems are of equal value.

Problem 1:
(Provide an answer to questions *1.* through *10*)

1. Function Φ_1 is C^0 continuous and function Φ_2 is C^1 continuous over a 1D element. Draw $\Phi_1(x)$, $\frac{d\Phi_1}{dx}$ and $\Phi_2(x)$, $\frac{d\Phi_2}{dx}$

2. Shape function of C^0 element satisfy the relation $\sum N_i = 1$, but such is not the case for shape functions of C^1 element (such as plane beam), explain why?

3. In plain problems, the compatibility equation is: $\epsilon_{x,yy} + \epsilon_{y,xx} = \gamma_{xy,xy}$. Explain the concept of the compatibility conditions of the strain field within a deformed solid.

4. Determine if the following stress field is a valid solution of a plane elasticity problem: $\sigma_x = 3a_1x^2y$, $\sigma_y = a_1y^3$ and $\tau_{xy} = -3a_1xy^2$, where a_1 is a constant. The solid is isotropic elastic and the body forces are zero.

5. Draw the approximate shape functions for the two truss elements shown in figure 1-a and 1-b.

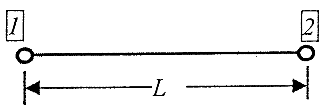

Figure 1-a

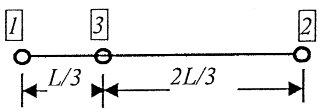

Figure 1-b

6. What is the physical interpretation of the Jacobian matrix's determinant, det[J], at a given integration point within a Q4 element?

7. How many non-zero eigenvalues has the stiffness matrix of a square bilinear element in plane strain condition?

8. What are the advantages of using the <u>reduced integration</u> technique when calculating the stiffness matrices of some elements to solve stress problems?

9. Explain the <u>h-refinement technique</u> to secure a convergent stresses from a finite element analysis.

10. Give two techniques commonly used to model embedded rebars in reinforced concrete structures.

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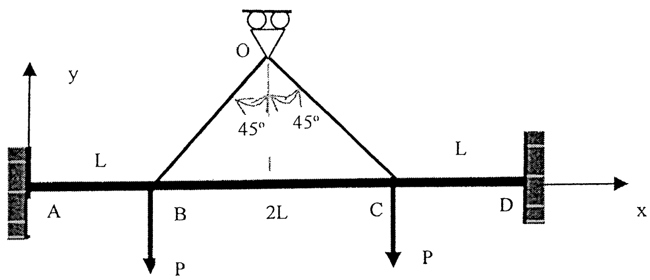
Problem 2

Elements AB, BC and CD, shown in Figure 2, represent a beam with constant inertia I. The two elements OB and OC are two truss members with cross-section area A. Both the beam and truss elements are made from the same material with an elastic modulus E.

Find (a) the displacement and rotations at all nodal points, (b) the shear forces diagram and (c) the bending moment diagram. Use $EA = \frac{EI}{6L^2}$



(Figure. 2)

The stiffness matrix of the beam element is shown below.

$$[k] = \frac{EI}{L}\begin{bmatrix} \frac{12}{L^2} & \frac{6}{L} & -\frac{12}{L^2} & \frac{6}{L} \\ \frac{6}{L} & 4 & -\frac{6}{L} & 2 \\ -\frac{12}{L^2} & -\frac{6}{L} & \frac{12}{L^2} & -\frac{6}{L} \\ \frac{6}{L} & 2 & -\frac{6}{L} & 4 \end{bmatrix}$$

Problem 3

Determine the stresses in a thin triangular plate (Figure 3) subjected to a temperature rise of 30°C. The plate is modeled with only *one triangular* element as shown in figure 3. The height of the plate is *45 mm* and the base width is *25 mm*. Other numerical data are given below:

thickness = *1 mm*; E = 30, 000 *N*/mm^2; $\nu = 0.25$; $\alpha = 11\times10^{-6}\,\dfrac{m}{m\,^oC}$

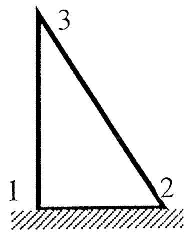
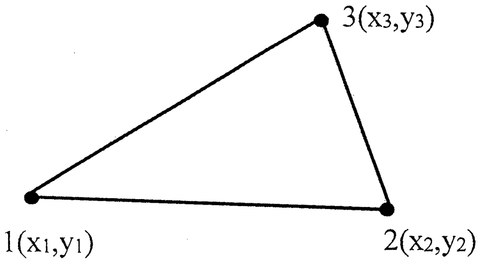


Figure 3

Note: The strain-displacement and the stiffness matrices of triangular element is given in the Appendix A.

Appendix A
Stiffness matrix of a CST element (6 dof)



$$[B]=\frac{1}{2A}\begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$\frac{t}{4A}\begin{bmatrix} H_1y_{23}^2+Gx_{32}^2 & H_2x_{32}y_{23}+Gx_{32}y_{23} & H_1y_{31}y_{23}+Gx_{32}x_{13} & H_2x_{13}y_{23}+Gx_{32}y_{31} & H_1y_{12}y_{23}+Gx_{21}x_{32} & H_2x_{21}y_{23}+Gx_{32}y_{12} \\ & H_1x_{32}^2+Gy_{23}^2 & H_2x_{32}y_{31}+Gx_{13}y_{23} & H_1x_{32}x_{13}+Gy_{31}y_{23} & H_2x_{32}y_{12}+Gx_{21}y_{23} & H_1x_{32}x_{21}+Gy_{12}y_{23} \\ & & H_1y_{31}^2+Gx_{13}^2 & H_2x_{13}y_{31}+Gx_{13}y_{31} & H_1y_{12}y_{31}+Gx_{13}x_{21} & H_2x_{21}y_{31}+Gx_{13}y_{12} \\ & \mathit{Symmetry} & & H_1x_{13}^2+Gy_{31}^2 & H_2x_{13}y_{12}+Gx_{21}y_{31} & H_1x_{13}x_{21}+Gy_{12}y_{31} \\ & & & & H_1y_{12}^2+Gx_{21}^2 & H_2x_{21}y_{12}+Gx_{21}y_{12} \\ & & & & & H_1x_{21}^2+Gy_{12}^2 \end{bmatrix}$$

$$H_1=\frac{2G(1-a\nu)}{(1-\nu-a\nu)},\quad H_2=\frac{\nu H_1}{1-a\nu},\quad G=\frac{E}{2(1+\nu)},\quad x_{ij}=x_i-x_j\ \ y_{ij}=y_i-y_j$$

a=0 plane stress, *a=1* plane strain
A is the area of the triangle
t is the thickness of the triangle