National Exams May 2018
04-BS-1, Mathematics
3 hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to include a clear statement of any assumptions made along with their answer.
2. One of two calculators is permitted - any Casio or Sharp approved model. This is a CLOSED BOOK exam. However, candidates are permitted to bring ONE AID SHEET written on both sides.
3. Any five questions constitute a complete paper. Only the first five questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Marking Scheme:

1. 20 marks
2. (a) 10 marks, (b) 10 marks
3. (a) 10 marks, (b) 10 marks
4. 20 marks
5. 20 marks
6. 20 marks
7. 20 marks
8. 20 marks

1. Find the general solution of the differential equation

$$x^2y'' - 4xy' + 6y = 3x^4.$$

Note that $'$ denotes differentiation with respect to x.

2. Solve the following initial value problems:

(a) $y' + 2ty^2 = 0, \ \ y(1) = 2,$

(b) $y'' - y' - 2y = 3t^2, \ \ y(0) = 0, \ \ y'(0) = 0.$

Note that in each case, $'$ denotes differentiation with respect to t.

3. Let $f(x, y, z) = x^2 + y^2 + z^2 + 2y - 3x$, and let $g(x, y, z) = 3x + y^2 - z^2$.

(a) Find an equation for the tangent plane to the surface $g(x, y, z) = 9$ at the point $(3, -1, 1)$.

(b) Find the line tangent to the intersection of the surfaces $f(x, y, z) = 0$ and $g(x, y, z) = 9$ at the point $(3, -1, 1)$.

4. Find the general solution to the following system of differential equations.

$$\frac{dx}{dt} = 4x - 18y,$$
$$\frac{dy}{dt} = -3x + y + e^{-5t}.$$

5. At what angle does the line represented parametrically by $x = 1 - t, \ \ y = t, \ \ z = 2 + 3t$ intersect the surface $z = 4 - x^2 + y^2$? You may leave your answer as an inverse sine or cosine.

6. Let C be the curve formed by the intersection of the cylinder $x^2 + y^2 = 9$ and the plane $z = 1 + y - 2x$, and let $\mathbf{v}$ be the vector function $\mathbf{v} = 4z\mathbf{i} - 2y\mathbf{j} + 2y\mathbf{k}$. Evaluate the line integral $\oint_C \mathbf{v} \cdot d\mathbf{r}$. Assume a clockwise orientation for the curve when viewed from above.

7. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = yz\mathbf{i} - 2xy\mathbf{j} + 3z\mathbf{k}$ and S is the surface of the region bounded above by the paraboloid $z = 4 - x^2 - y^2$ and below by the plane $z = 0$.

8. Find the minimum value of the function $F(x, y, z) = x - y + 2z$ subject to the constraint $x^2 + 3y^2 + 2z^2 = 5$.