National Exams – December 2018

16-Mec-A6 Advanced Fluid Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any approved Sharp/Casio calculator.
   The exam is OPEN BOOK.
3. Any FIVE (5) out of the 6 questions constitute a complete exam paper for a total of 100 MARKS.
   The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value (20 marks) and question items are marked as indicated.
5. Clarity and organization of the answer are important.

Marking Scheme:
1. 20 marks total (5 marks each item)
2. 20 marks total (5 marks each item)
3. 20 marks total ((a) 7 marks; (b) 7 marks; (c) 6 marks)
4. 20 marks total (5 marks each item)
5. 20 marks total
6. 20 marks total (5 marks each item)

(20) **Question 1**

A pressurized air canister has an internal volume of 0.5 m^3. The air (γ=1.4, R=287 J/(kg K), C_p=1,004.5 J/(kg K)) inside the canister is kept at a constant temperature of 300 K. Initially, the air inside the canister is pressurized to 10.0 MPa. The canister outlet consists of a valve shaped like a convergent-divergent nozzle. The throat area is given as A_T=1 mm^2 and the exit area as A_E=3.0 mm^2. The valve is opened to allow air to flow through. The air exhaust to atmosphere (P_b=100 kPa). It can be assumed that frictional losses are negligible. It can also be assumed that the air is still inside the canister (before entering the valve section).

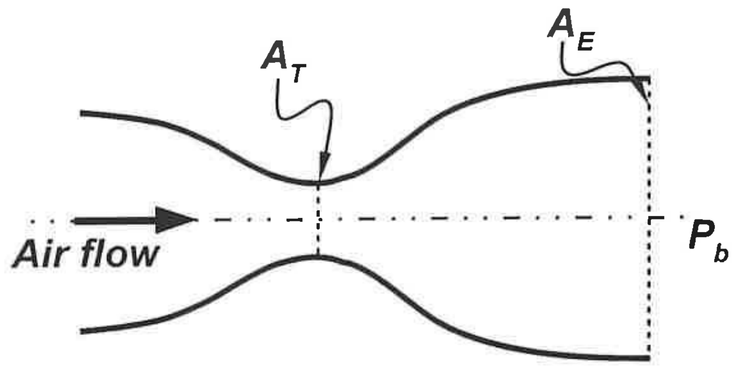


Figure 1: Cross-sectional schematic of valve section.

(5) (a) Determine the flow rate, exit temperature and speed of the air immediately after opening the valve (i.e. when the internal pressure is 10.0 MPa).

(5) (b) Determine the flow rate, exit temperature and speed of the flow when the internal pressure of the canister reaches 5.0 MPa.

(5) (c) At what internal pressure does a shock form exactly at the exit of the convergent-divergent nozzle? What is the flow rate at this point? What is the air temperature and speed immediately after the exit?

(5) (d) Below what canister pressure is the valve (nozzle) no longer choked?

(20) **Question 2**

A long municipal discharge pipe can be modelled as a source of strength m. The pipe is placed at a distance b from the bottom of a deep containment tank as shown in Figure 2. The city engineers want to drain the tank, but do not want to interrupt the discharge during the operation. The drain can be modelled as a sink of strength $-2m$ placed at the centre of the tank directly below the pipe, as shown in the figure. The city supervisor is concerned about the forces generated on the pipe. Your consulting company is asked to estimate these forces using potential flow theory.

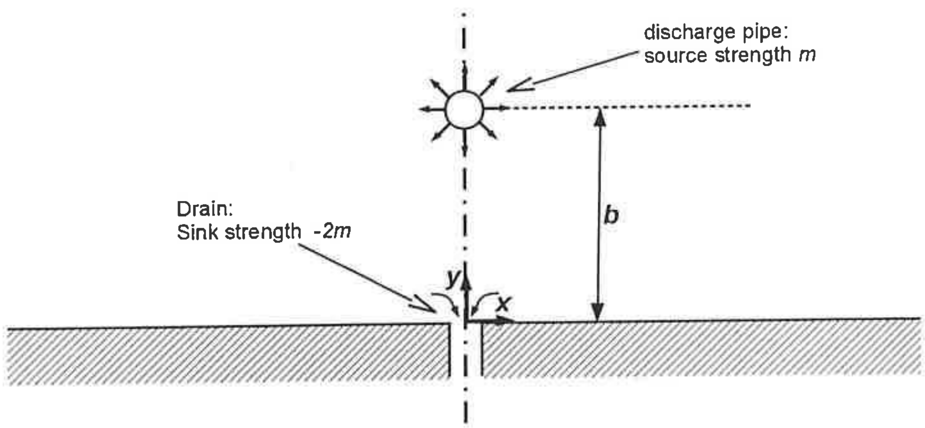


Figure 2: Cross-sectional view of discharge pipe next to a tank bed with a drain.

Assuming that the waste water density is the same as the water in the discharge tank, that the tank bed is flat, that the fluid velocity far from the discharge is negligible, and that the free surface effects can be neglected, determine:

(5) (a) The stream function that will represent this flow.

(5) (b) Verify that the tank bed is correctly simulated.

(5) (c) The velocity distribution along the tank bed.

(5) (d) The forces acting on the discharge pipe.

(20) **Question 3**

An oil (specific gravity = 0.9) issues from the pipe at the bottom of a large tank, as seen in the figure, at $Q = 1.0\ \text{m}^3/\text{h}$. The draining pipe has a diameter of $D = 1.3$ cm and a length of $L = 2$ m. The depth of oil in the tank is $h = 3$ m.

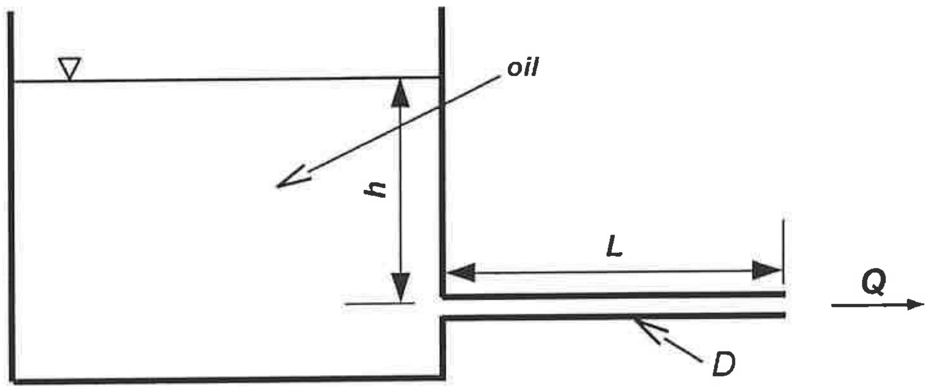


Figure 3: Tank with draining pipe.

(7) (a) What is the head loss through the draining pipe?

(7) (b) What is the kinematic viscosity of the oil in m^2/s?

(6) (c) Is the flow laminar?

(20) **Question 4**

Consider the combined gravity-Couette driven flow between two inclined, parallel plates, as depicted in the figure. The lower plate is stationary, while the upper plate moves at constant upward velocity V_{top}. The plates are very long and wide and they are separated by a small gap of length h. The fluid has viscosity μ and density ρ.

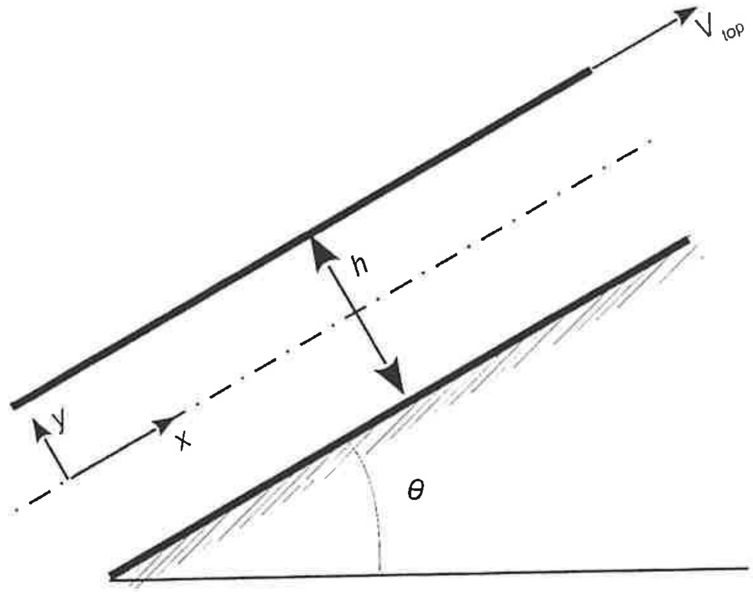


Figure 4: Inclined gravity and Couette driven flow.

(5) (a) What assumptions can be made about this flow scenario?

(5) (b) Write the complete Navier-Stokes equations governing this flow, including known Boundary Conditions.

(5) (c) Simplify and reduce the governing equations to a second order ordinary differential equation (ODE). Justify your assumptions.

(5) (d) Find the solution to this ODE using appropriate Boundary Conditions.

(20) **Question 5**

The lift force F on a missile is a function of its length L, velocity V, diameter D, angle of attack α, density ρ, viscosity μ, and speed of sound of the air c.

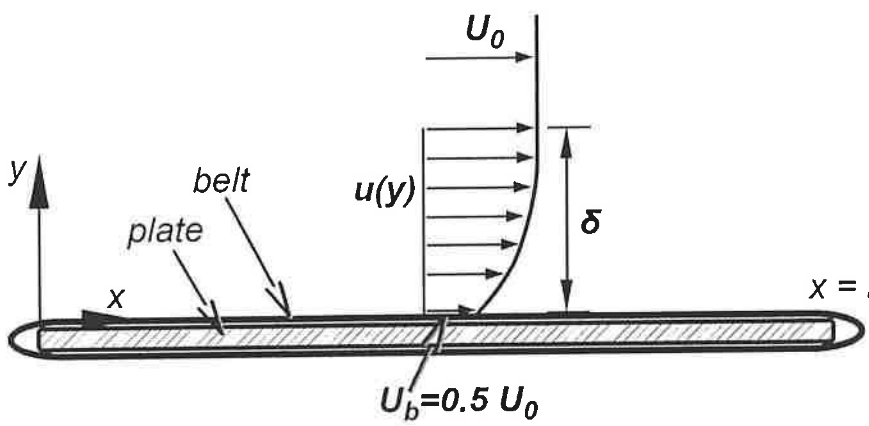
Find the dimensionless π groups and rewrite the function in terms of known π groups, if there are any.

(20) **Question 6**

A plate with a rolling belt around it (width W and length L) is exposed to a uniform free stream flow of constant speed U_0. The fluid is Newtonian with constant properties (ρ is the density, μ is the dynamic viscosity, and $\nu = \mu/\rho$ is the kinematic viscosity).

A laminar boundary layer develops over the belt and causes the belt to move. The belt moves towards the right (in the same direction as the free stream) at a constant speed of $U_b = 0.5\,U_0$, as shown in the figure. The plate and belt are very wide compared to the boundary layer thickness, δ, such that $W/\delta \gg 10$.

It is desired to find an approximation of the friction of the belt using the integral boundary layer equations. It can be assumed that at $x = 0$, $\delta = 0$. It can also be assumed that the free stream flow is irrotational.



(5) (a) Given the trial function of the form:

$$\frac{u}{U_0} = a + b\frac{y}{\delta} + c\left(\frac{y}{\delta}\right)^2 ,$$

determine suitable values for the coefficients a, b, and c.

(5) (b) Determine the expression for the boundary layer thickness, δ, as a function of the position x.

(5) (c) What is the local wall shear stress coefficient $C_{fx} = \tau_w/(\rho U_0^2/2)$?

(5) (d) What is the average wall shear stress, $\overline{\tau_w}$, between $x = 0$ and $x = L$?