National Exams May 2017

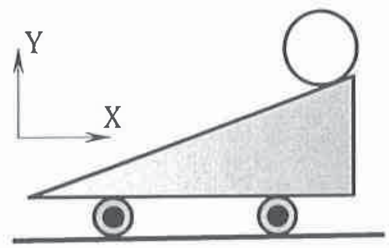
98-Phys-A1, Classical Mechanics

3 hours duration

NOTES:

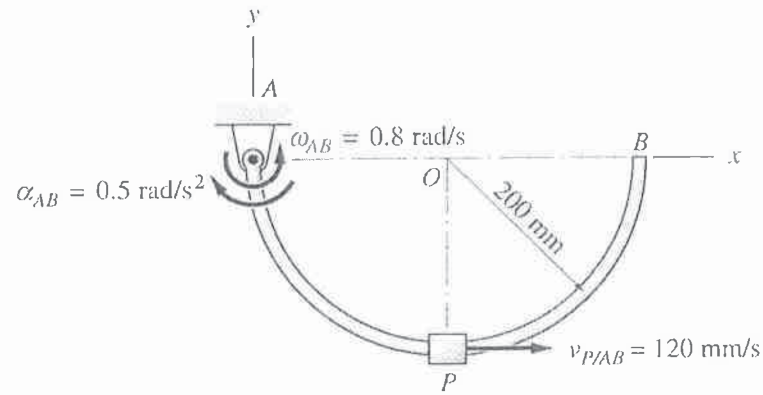
1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. This is a CLOSED BOOK EXAM.
   One of two calculators is permitted any Casio or Sharp approved models.
3. The exam consists of six questions; FIVE (5) questions constitute a complete exam paper. The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value.
5. Most questions require an answer in essay format. Clarity and organization of the answer are important.

1. A 2-*kg* sphere of 1-*m* diameter was initially at rest on top of a 5-*kg* wedge-shape surface with a 30-degree angle; see Figure. The surface is free to roll on a horizontal ground. Assume the motion maintains its plane-of-motion in the X-Y plane shown in the Figure. No friction should be considered and neglect the mass of the wheels of the surface. Answer the following questions. (a) Is the constraint of motion holonomic or non-holonomic as it rolls down the slope? Provide your reasons. (b) How many degrees-of-freedom are needed to describe the motion of the sphere? (c) Use the Lagrange's equations to find the force of the constraint. Note, the moment of inertia of a uniform sphere rotating about its mass center is $2mr^2/5$ where m is the mass and r is the radius of the sphere.

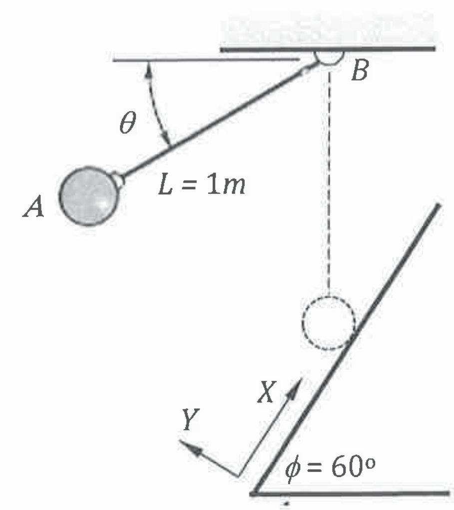


2. Consider a particle of mass m and velocity $(d/dt)\vec{x}$ in a central force field described by a potential function $U(|\vec{x}|)$ where $\vec{x}$ is the position vector of the particle and $|\cdot|$ is the norm of the vector. Let $\vec{y} = R\vec{x}$ where R is a rotation. (a) Show that the Lagrangian L is invariant under any rotation; i.e., the Lagrangian written in $\vec{y}, \vec{x}$ is the same. (b) Assume the particle rotates with an angular velocity $\vec{\omega}$ about the axis L. Show that the angular momentum of the particle about L is invariant. (c) Explain (b) in the context of Noether's theorem.

3. The collar P slides along a semicircular rod AB of radius 0.2 m. The rod rotates about point A with a counter-clockwise angular velocity 0.8 rad/s and a clockwise angular acceleration 0.5 rad/s^2. The speed of the collar P is kept constant relative to the semicircular at 0.12m/s. Determine the absolute velocity and acceleration of the collar at the instant shown.



4. A 2 kg mass attached to a 1 m long string swings down from an angle of $\theta = 0^\circ$ and impact with a $\phi = 60^\circ$ inclined surface; see the right-hand figure. At the time of impact, the string is exactly vertical. Assume the coefficient of restitution $e = 0.7$ and ignore the mass of the string. You must answer this question using impulse momentum principle.



(a) Consider the initial state to be right before the mass impact with the incline, and the final state to be right after the mass impact with the incline. Write down the linear impulse momentum equations for the mass in the X-Y coordinate system shown in the figure.
(b) Find the velocity of the mass right after the impact.
(c) Assume the impact lasted for 0.05 second. Find the averaged impact force.
(d) Would your answer for (a) change if the string is replaced by a rigid rod? If so, why?

5. Assume the angular velocity and the inertia tensor of a rigid body, $\vec{\omega} = \omega_x \vec{\imath} + \omega_x \vec{\jmath} + \omega_x \vec{k}$, and $\boldsymbol{I}$, respectively. Let I_{xx}, I_{yy}, I_{zz} be the principal moments of inertia derived from $\boldsymbol{I}$. (a) Assume the rigid body satisfies the condition $I_{xx} = I_{yy} = I_{zz}$, and an impulsive moment $\vec{M}(t) = \vec{M}_0 \delta(t)$ is applied to the body where $\vec{M}_0 = a\vec{\imath} + b\vec{\jmath} + c\vec{k}$ is a fixed vector with a, b, $c > 0$, and $\delta(t)$ is the Dirac delta function. Discuss how the rotation will progress for $t > 0$ and show your analysis. (b) Assume the condition $I_{xx} > I_{yy} > I_{zz}$ and $\vec{M}(t) = \vec{M}_0 \delta(t) = (a\vec{\imath} + 0\vec{\jmath} + c\vec{k})\delta(t)$ with a, $c > 0$. Discuss how the rotation will progress for $t > 0$ and show your analysis.

6. At $t = 0$, the cable attached to the 1-*kg* rigid rod at point B snaps; see Figure. Assume the spring immediately detaches from the rod for $t > 0$. Determine the angular velocity and the acceleration at the center of mass of the rod AB after one second. Note the moment of inertia of a uniform bar rotating about its mass center is $ml^2/12$ where l is the length of the bar and m is the mass of the bar.