PROFESSIONAL ENGINEERS OF ONTARIO

ANNUAL EXAMINATIONS – December 2014

07-Mec-B10 Finite Element Analysis

3 hours duration

INSTRUCTIONS:

1. **If doubt exists as to the interpretation of any of the questions, the candidate is urged to submit a clear statement of the assumption(s) made with the answer.**

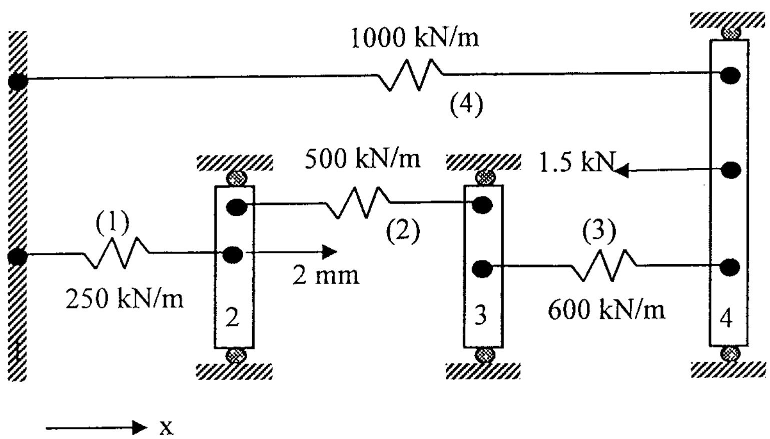
2. **This examination paper is open book; candidates are permitted to make use of any textbooks, references or notes.**

3. **Any non-communicating calculator is permitted. Candidates must indicate the type of calculator(s) that they have used by writing the name and model designation of the calculator(s) on the first inside left hand sheet of the first examination workbook.**

4. **Candidates are required to attempt any five questions. The questions are to be solved within the context of the finite element method.**

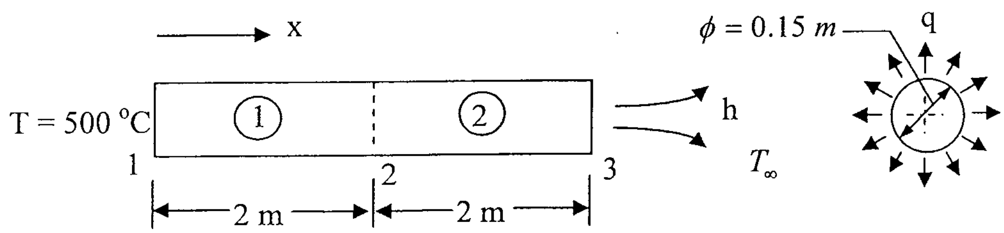
5. **The questions are equally weighted. Indicate which five questions are to be marked on the cover of the first examination workbook.**

Question 1. [20 marks]



For the spring assemblage shown in the above figure,
(a) [6 marks] Write the total potential energy expression of the system
(b) [6 marks] Derive the system equilibrium equations using the expression obtained in part (a) in the Principle of Minimum Potential Energy
(c) [4 marks] Write the system equilibrium equations in matrix format.
(d) [2 marks] Determine the nodal displacements and reaction forces.
(e) [2 marks] Determine the forces in element #2 only.

Question 2. [20 marks] The figure below depicts a 4 m long rod with diameter $\phi = 0.15$ m. The rod is discretized into two elements of equal length numbered 1 and 2 as shown.



The thermal conductivity in the x-direction of the first and second element is $K_{xx}^{(1)} = 300$ W/(m.°C) and $K_{xx}^{(2)} = 600$ W/(m. °C), respectively. A uniform internal heat source $Q^{(1)} = 400$ W/m^3 is located in the first element and that in second element is $Q^{(2)} = 500$ W/m^3. Further, a uniform outgoing heat flux $q_{out} = 10$ W/m^2 acts over the whole cylindrical surface of the rod.

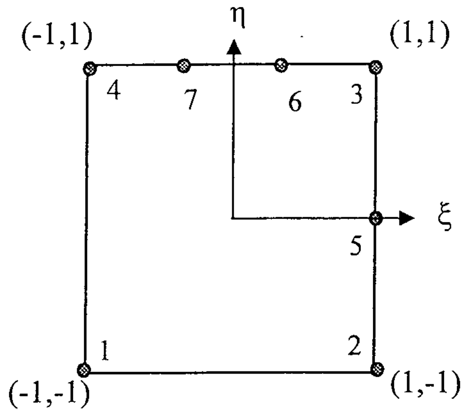
The temperature at the left-hand end of the rod is maintained constant at 500 °C. Heat convection arises **only** at right-hand end of the rod with convection heat flux $h = 30$ W/(m^2. °C) and the downstream temperature $T_\infty = 100$ °C. Determine

(a) [14 marks] the temperature at nodes 2 and 3,

(b) [3 marks] the heat flux over element #1,

(c) [3 marks] the heat flow rate and the direction at node1.

Question 3. [20 marks]



(a) [14 marks] Determine the shape functions (N_i, $i = 1$ to 7) of the seven-node transition element in natural/local coordinates (ξ, η) such that $-1 \le \xi, \eta \le 1$.

(b) [4 marks] Evaluate the shape function N_3 at the sixth node and the centroid of the element.

(c) [2 marks] Assume the field variables to the problem are displacement components denoted by u and v for the ξ and η directions, respectively. If the nodal displacement components are zero except $u_3 = u_4 = u_6 = u_7 = 0.025$ mm, determine an expression for the field variables in the natural/ local coordinates (ξ, η).

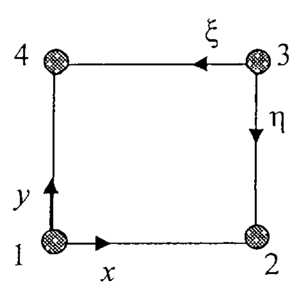
Question 4. [20 marks]

(a) [4 marks] Briefly explain in a sentence or two the difference between basis function and shape function.

(b) [4 marks] Briefly explain the meaning of geometric isotropy in a sentence or two.

(c) [4 marks] State the **two** properties that a polynomial representation of a field variable variation in an element should have to ensure that the element has geometric isotropy?

(d) [8 marks] Consider the square element below for which the field variable u is interpolated in

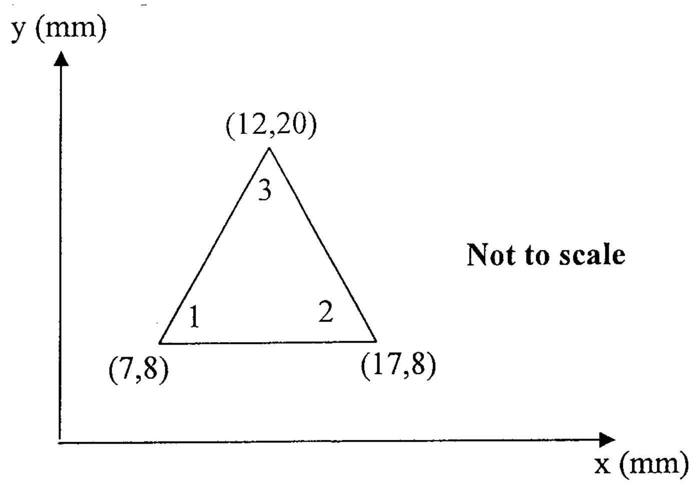


the Cartesian (x,y) coordinate axis centred at node 1 as

$$u(x,y) = C_1 + C_2x + C_3y + C_4xy$$

If the length of each side of the element is L, use the (ξ,η) coordinate axis centred at node 3 (as depicted in the schematic of the element) to show that the element has geometric isotropy.

Question 5. [20 marks]



Not to scale

The nodal displacements for the plane strain element shown in the figure above are:

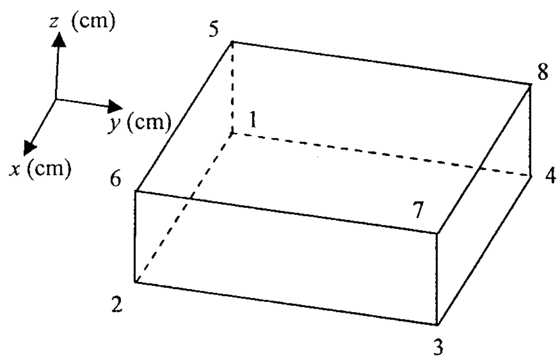
$u_1 = 0.004$ mm and $v_1 = 0.002$ mm; $u_2 = v_2 = 0.0$ mm; $u_3 = 0.005$ mm and $v_3 = 0.0$ mm

The plate thickness $t = 0.8$ mm, and it is made from a material with Young's modulus $E = 210$ MPa and Poisson's ratio $\nu = 0.3$.

(a) [15 marks] Determine the element stresses σ_x, σ_y, and τ_{xy}.

(b) [5 marks] Determine the principal stresses σ_1 and σ_2 and principal angle θ_p.

Question 6. [20 marks]



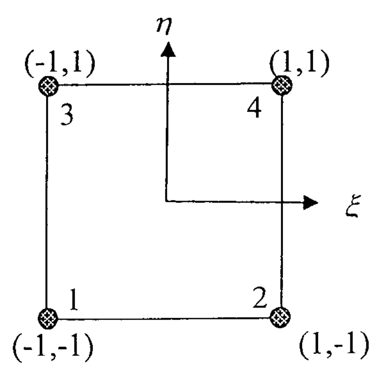
1: (2,3,2)
2: (7,3,2)
3: (7,9,2)
4: (2,9,2)
5: (2,3,4)
6: (7,3,4)
7: (7,9,4)
8: (2,9,4)

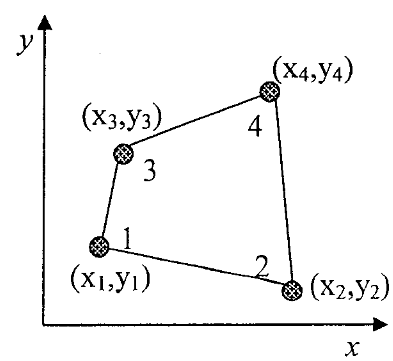
Not to scale

(a) [4 marks] Determine the shape functions (N_i, i = 1 to 8) of an eight-node hexahedron element in natural/local coordinates (ξ, η, ζ) such that $-1 \leq \xi, \eta, \zeta \leq 1$. The node numbering is identical to that shown in the above representative global element.
(b) [3 marks] Evaluate the shape function N_3 at the third node, the fifth node, and the centroid of the element.
(c) [3 marks] Assume the field variables to the problem are displacement components denoted by $u, v,$ and w in the ξ, η, and ζ directions, respectively. If the nodal displacement components are zero except $v_3 = v_4 = v_7 = v_8 = 0.025$ mm, compute the field variables in the natural/ local coordinates (ξ, η, ζ).
(d) [7 marks] Determine the Jacobian matrix and evaluate the Jacobian of the above element.
(e) [3 marks] Determine the normal strain $\varepsilon_y = \frac{\partial v}{\partial y}$ and shear strain $\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ at the centre of the element.

Question 7. [20 marks]

(a) [2 marks] What is an isoparametric element?
(b) [10 marks] The four-node isoparametric quadrilateral element shown below is used to map a region in the parent domain into that in the global domain.



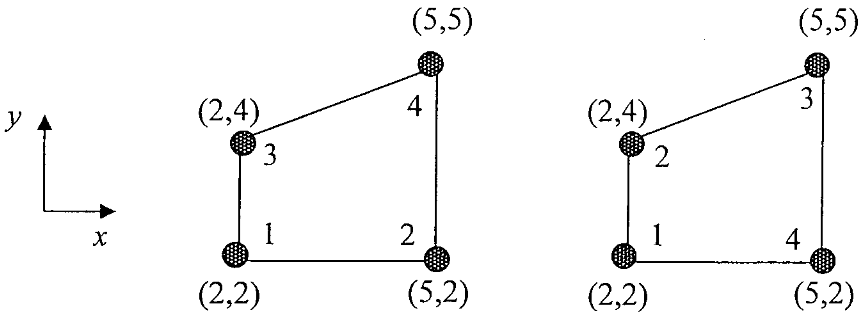


The shape functions of the element are given as

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),\ N_2 = \frac{1}{4}(1+\xi)(1-\eta),\ N_3 = \frac{1}{4}(1-\xi)(1+\eta),\ N_4 = \frac{1}{4}(1+\xi)(1+\eta)$$

Determine the Jacobian matrix of the element.

(c) [4 marks] Use the Jacobian matrix obtained in (b) to evaluate the Jacobian of the following elements.



(i) (ii)

(d) [4 marks] What can be concluded from the Jacobian expressions obtained in (c) in light of the mapping of the coordinates?