National Exams December 2018

16-Elec-A2, Systems & Control

3 hours duration

NOTES:

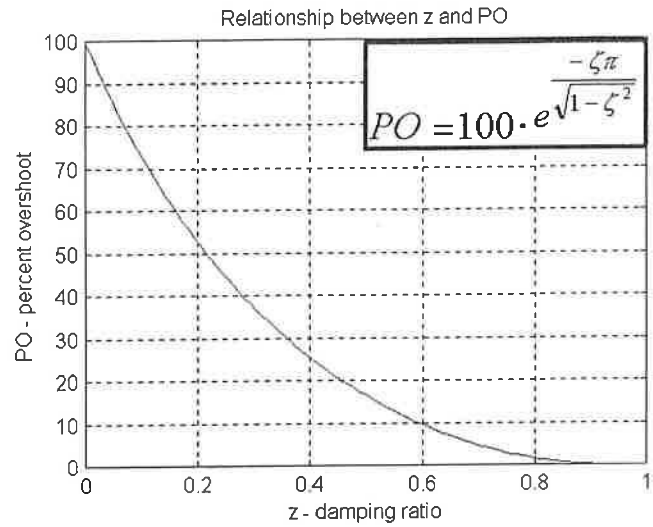
1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, **handwritten**, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet must be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate the questions which should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

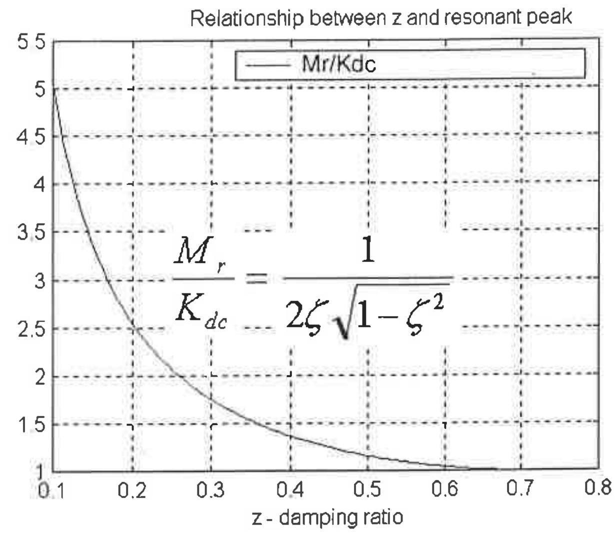
A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1-e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos at \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1 - e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t + \cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

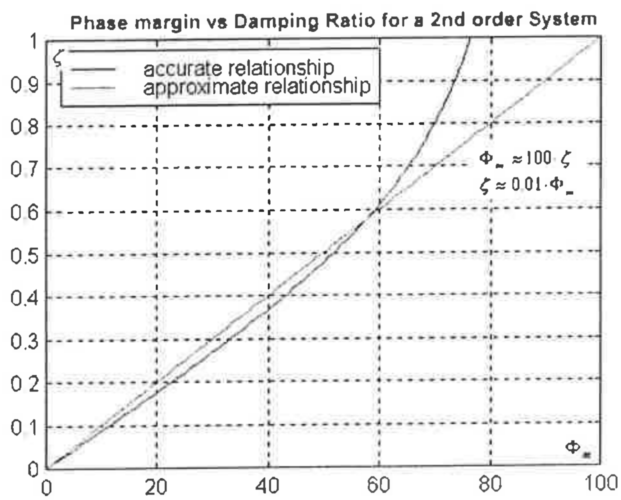
Useful Plots & Formulae



PO vs. Damping Ratio



Resonant Peak vs. Damping Ratio



Second Order Model:

$$G_m(s) = K_{dc}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ – Damping Ratio (zeta), of the model

ω_n – Frequency of Natural Oscillations of the model

K_{dc} – DC Gain of the model

Definitions for Controllability Matrix, $\mathbf{M_c}$, and Observability Matrix, $\mathbf{M_o}$:

$$\mathbf{M}_c = [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B}] \qquad \mathbf{M}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix}$$

Definition for Transfer Function from State Space:

$$G(s) = \mathbf{C} \cdot (s\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{B} + \mathbf{D}$$

Question 1 (Compulsory)

Controller Design by Pole Placement, Response Specifications, Second Order Model.

Consider a closed loop positioning control system working in a configuration shown in Figure Q1.1 that could become either a Lead or a Lag Control, depending on the choice of its parameters.

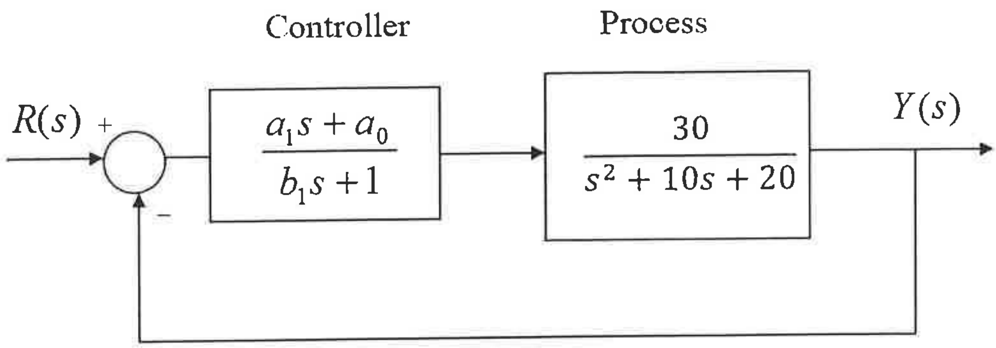


Figure Q1.1

Design requirements are:

- Steady State Error for the unit step input for the compensated closed loop system is to be equal to **one-fourth** of the uncompensated system Steady State Error.
- The compensated Closed Loop step response of this system is to have the following specifications: $PO = 10\%$ and $T_{settle(\pm 2\%)} = 3$ sec.

1) **(4 marks)** Derive the Closed Loop system transfer function in terms of Controller parameters a_0, a_1 and b_1, and write the system Characteristic Equation, $Q(s) = 0$.

2) **(5 marks)** Determine the Closed Loop system damping ratio, ζ and the Closed Loop system frequency of natural oscillations, ω_n to meet the transient response requirements. Next, determine the value of the Controller DC gain, a_0 to meet the Closed Loop system steady state error requirement.

3) **(6 marks)** Create a "desired" third order Characteristic Equation, that includes the "dominant" poles, and a third real pole at a to-be-determined location ($s_3 = -d$ where d is an unknown variable), then match it with the coefficients of the actual third order system Characteristic Equation to find the two unknown Controller parameters, a_1 and b_1 as well as the location of the third closed loop pole, d.

4) **(5 marks)** Based on the results, discuss the following points:

   a. Is the resulting Controller configuration a Lead Controller or a Lag Controller?
   b. Is the third closed loop pole at $s_3 = -d$ in the insignificant region of the complex plane, as compared to the location of the "dominant" poles?
   c. Is the resulting closed loop zero in the insignificant region of the complex plane, as compared to the location of the "dominant" poles?

Question 2 (Compulsory)

Transfer Functions - Signal Flow Diagrams – Mason's Gain Formula. Stability – Routh Array and Routh-Hurwitz Criterion of Stability. Steady State Errors.

Consider the block diagram of the servo-control system for one of the joints of a robot arm, shown in Figure Q1.1, where the input is the reference angular velocity (speed) for the robot arm, $R(s)$, the output is the actual load velocity of the arm, $Y(s)$, and the forward path contains a Proportional + Integral (PI) Controller, a calibration gain, motor and robotic arm dynamics and a gearbox. The Proportional + Integral (PI) Controller is described as shown and has a time constant $\tau_i = 2.0$ seconds. Two transfer functions have been defined, the closed loop system transfer function $G_{cl}(s)$, between the actual angular load velocity and the reference angular velocity, and the disturbance transfer function, $G_d(s)$, between the Torque disturbance signal, $D(s)$, and the load velocity, $Y(s)$:

$$G_{cl}(s) = \frac{Y(s)}{R(s)} \qquad G_{dist}(s) = \frac{Y(s)}{D(s)}$$

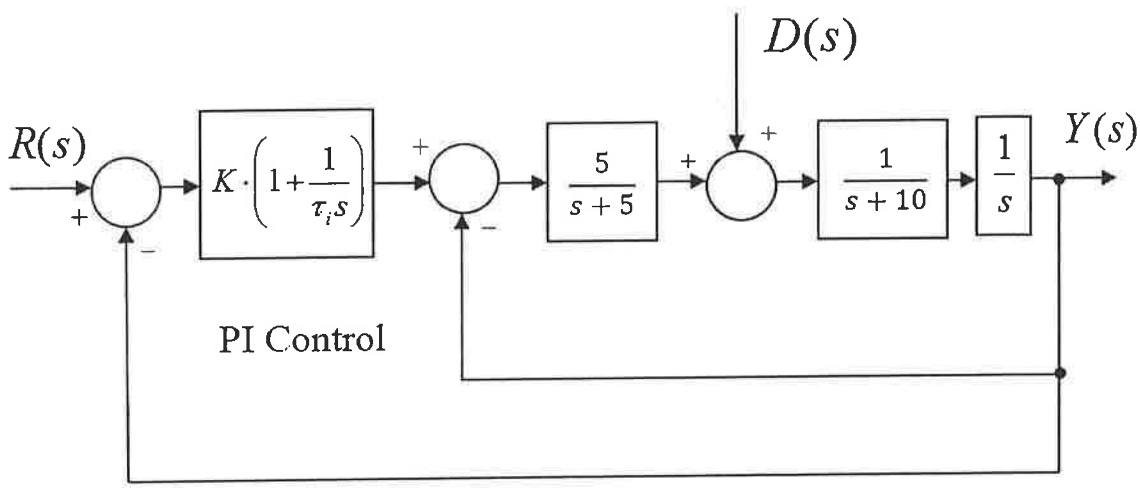


Figure Q2.1: Servomotor Positioning System

1) **(6 marks)** Find $G_{cl}(s)$ and write it in the TF format (polynomial ratio), as a function of the Proportional gain, K.

2) **(4 marks)** Find $G_{dist}(s)$ and write it in the TF format (polynomial ratio), as a function of the Proportional gain, K.

3) **(10 marks)** Determine the range of the Proportional Controller gains, K, for a safe, stable operation of the closed loop system. Specify the critical value(s) of the Gain, K_{crit}, when the system is marginally stable, as well as the frequency of oscillations, ω_{osc}, resulting when $K = K_{crit}$.

Question 3

Second Order Dominant Poles Model, System Performance.

Consider, once again, the servo-positioning system under Proportional + Integral (PI) Control from Question 1, shown in Figure Q1.1. Assume no disturbance signal present. The Controller Operational Gain value was set to $K_{op} = 3.0$ and the closed loop transfer function, $G_{cl}(s)$, was found to be:

$$G_{cl}(s) = \frac{250(s+0.5)}{s^4 + 15s^3 + 50s^2 + 255s + 125} = \frac{250(s+0.5)}{(s+12.57)(s+0.54)(s^2+1.89s+18.47)}$$

1) **(4 marks)** What is the DC gain of the closed loop system, $G_{cl}(0)$?
2) **(8 marks)** Assume that a 2nd order dominant poles model (see page 3) will be applicable for the closed loop system. Determine the model parameters, K_{dc}, ζ, ω_n, and write the model transfer function, $G_m(s)$.
3) **(8 marks)** Evaluate the following unit step response specifications: Percent Overshoot (PO), Settling Time ($T_{settle\ \pm 2\%}$), Period of Oscillations (T_{period}), Rise Time ($T_{rise\ (0-100\%)}$) and the Steady State Error ($e_{ss(step)\%}$).

Question 4

Steady State Error Analysis.

Consider the same servo-positioning system under Proportional + Integral (PI) Control from Question 1, shown in Figure Q1.1.

1) **(5 marks)** What is the System Type? When the Proportional Gain is set to $K = 1$, calculate the Error Constants and the corresponding Steady State Errors: K_{pos}, $e_{ss(step\%)}$, K_v, $e_{ss(ramp)}$, K_a, $e_{ss(parab)}$.
2) **(5 marks)** The system is to exhibit zero steady state error to a unit step and the steady state error to a unit ramp is to be: $e_{ss(ramp)} = 0.04\ V/V$.
   a. Calculate the required operating value for the Proportional Gain (K_{op}).
   b. When $K = K_{op}$, will the system be still stable? Calculate the corresponding Gain Margin (G_m), in V/V units.
3) **(5 marks)** If the system is stable when $K = K_{op}$, re-calculate the Error Constants and the corresponding Steady State Steady State Errors: K_{pos}, $e_{ss(step\%)}$, K_v, $e_{ss(ramp)}$, K_a, $e_{ss(parab)}$.
4) **(5 marks)** Assume $K = K_{op}$ as calculated above. Assume the reference angular velocity and the disturbance torque signals to be described as shown below, and then calculate the total steady state error in the system response. HINT: Use Final Value Theorem.

$$r(t) = 2t \cdot 1(t) \qquad d(t) = 10t \cdot 1(t)$$

Question 5

Second Order Dominant Poles Model in s-Domain and in Frequency Domain (Open and Closed Loop), Step Response Specifications.

A unit feedback control system is working under a Proportional Controller, as shown in Figure Q5.1, with the Proportional Controller Gain value set at $K_p = 5$.

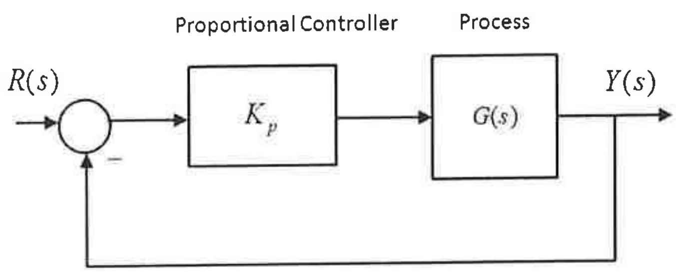


Figure Q5.1

The process transfer function is described as follows:

$$G(s) = \frac{20(s+3)}{(s+0.1)^2(s+20)^2}$$

$$G_{open}(s) = K_p \cdot G(s) \qquad G_{cl}(s) = \frac{G_{open}(s)}{1 + G_{open}(s)}$$

1) **(5 marks)** The closed loop transfer function of the system has one zero: $z_1 = -3$, and four poles: $p_1 = -0.19 + j0.86$, $p_2 = -0.19 - j0.86$, $p_3 = -17.83$, and $p_4 = -22.0$. Explain why the second order dominant poles model is appropriate for the closed loop transfer function, and derive its transfer function, $G_{m1}(s)$.

2) **(5 marks)** Determine the second order dominant poles model, $G_{m2}(s)$, based on the Closed Loop frequency response. The magnitude plot of $|G_{cl}(j\omega)|$ is shown in Figure Q5.2.

3) **(5 marks)** Determine the second order dominant poles model, $G_{m3}(s)$, based on the Open Loop frequency response. The frequency response plots of $G(j\omega)$ are shown in Figure Q6.2 – note that it does not include the Proportional Gain.

4) **(5 marks)** Use the model you consider the most accurate to estimate the following closed loop step response specifications: $e_{ss(step\%)}$, $T_{rise(0-100\%)}$, $T_{settle(\pm 2\%)}$ and PO.

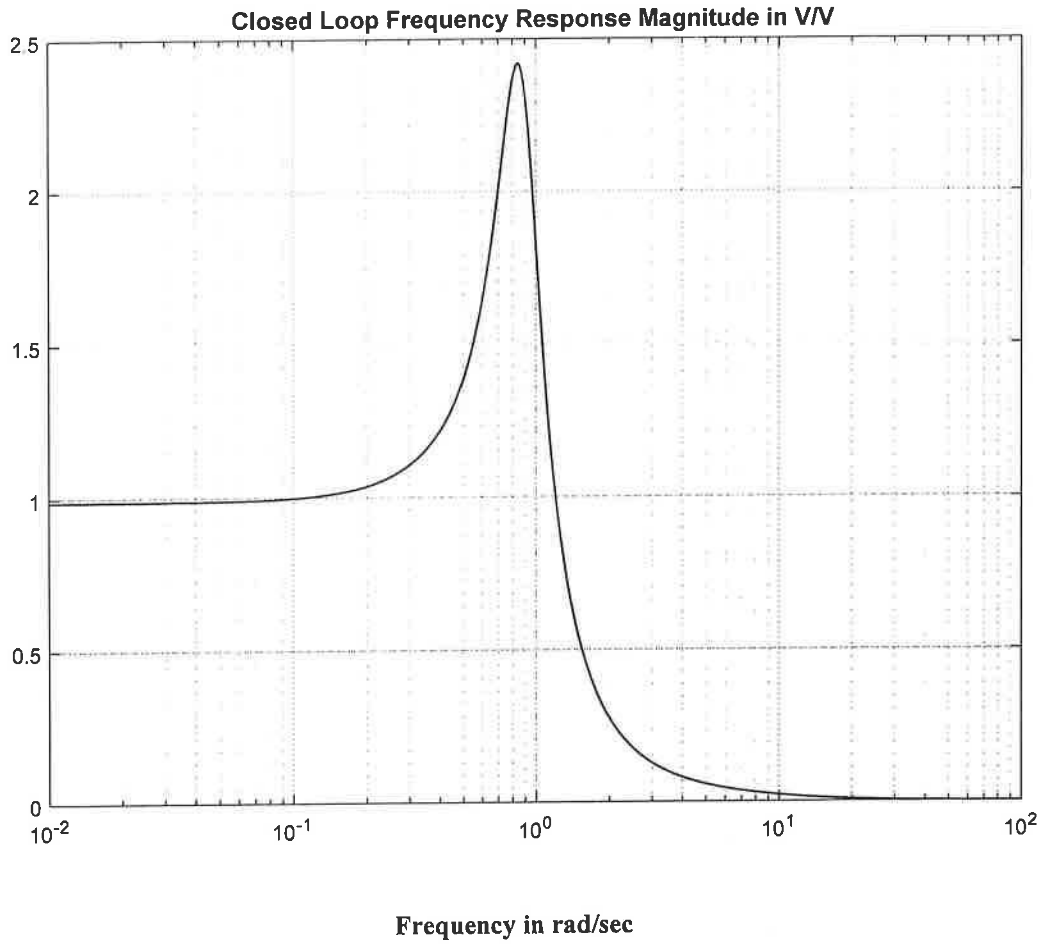


Figure Q5.2 – Closed Loop Frequency Response (Magnitude) Plot of $|G_{cl}(j\omega)|$

Question 6

Controller Design in Frequency Domain – Lag Controller, Step Response Specifications.

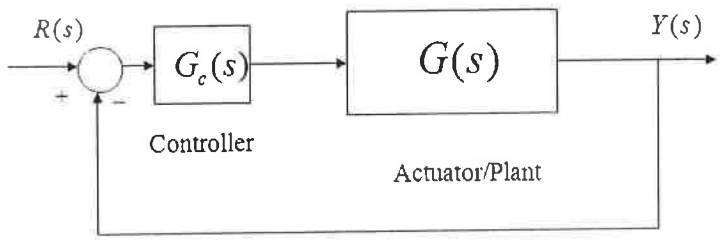


Figure Q6.1

Consider a unit feedback closed loop control system, as shown on the left.

The system is to operate under **Lag Control.** The Lag Controller transfer function is as follows:

$$G_c(s) = K_c \cdot \frac{\tau\alpha s + 1}{\tau s + 1} = \frac{a_1 s + a_0}{b_1 s + 1}$$

Where τ is the so-called Lag Time Constant and $\alpha < 1$.

The process transfer function G(s) is as follows:

$$G(s) = \frac{20(s+3)}{(s+0.1)^2(s+20)^2}$$

Frequency response plots of $G(j\omega)$ are shown in Figure Q6.2 - note that the transfer function G(s) in Question 6 is the same as in Question 5. Design requirements are:

- Steady State Error for the unit step input for the compensated closed loop system is to be **one fifth** of the Steady State Error for the uncompensated system
- Percent Overshoot of the compensated closed loop system is to be no more than 15%.

1) **(2 marks)** Calculate the Position Constant for the uncompensated system (K_{pos_u}), then the Position Constant for the compensated system (K_{pos_c}) that would meet the design requirements.
2) **(4 marks)** Read off the Phase Margin of the uncompensated system (Φ_{m_u}) and then decide what value of the Phase Margin for the compensated system (Φ_{m_c}) would meet the design requirements. Read off the crossover frequency of the uncompensated system (ω_{cp_u}) and then decide what value of the crossover frequency for the compensated system (ω_{cp_c}) would meet the design requirements.
3) **(10 marks)** Calculate the appropriate Lag Controller parameters and clearly write the Lag Controller transfer function $G_c(s)$.
4) **(4 marks)** Estimate the compensated closed loop step response specs: PO, $e_{ss(step\%)}$, $T_{rise(0-100\%)}$ and $T_{settle(\pm 2\%)}$.

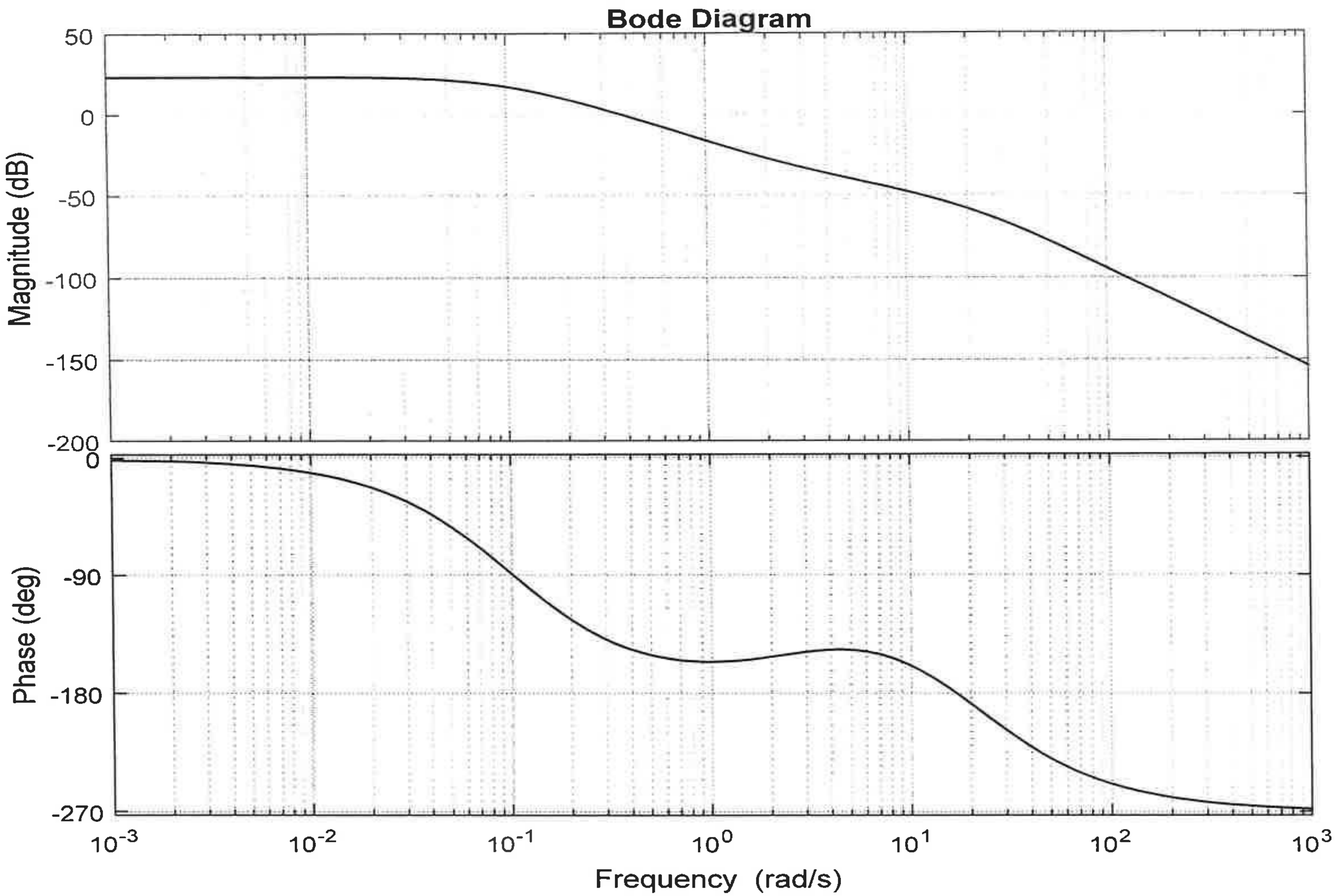


Figure Q6.2 - Frequency Response Plots of the Process $G(j\omega)$ in Question 5 and Question 6

Question 7

State Space Model from Transfer Functions, Pole Placement by State Feedback Method, Steady State Errors to Step and Ramp Inputs.

Consider a linear open loop system described by the following state space model:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-4 & 1\\ 1 & 0\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}\cdot u$$

$$y = [1 \quad 1]\cdot\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} + [0]\cdot u$$

1) (**4 marks**) Find the system eigenvalues. Is the open loop system stable?
2) (**4 marks**) Find the open loop system transfer function, $G_{open}(s) = \frac{Y(s)}{U(s)}$
3) (**4 marks**) Determine if the open loop system is observable and/or controllable.
4) (**4 marks**) Place the system in a closed loop configuration with the reference input **r** and assume the controller equation to be in the form:

$$u = K\cdot\left(r - \mathbf{k}^T\cdot\mathbf{x}\right)$$

Determine the values of the Proportional Gain K and the state feedback vector gains **k** so that the closed loop system will have poles at: -5 and -6, and the steady-state error to a step input will be zero.

5) (**4 marks**) Find the closed loop system transfer function, $G_{cl}(s) = \frac{Y(s)}{R(s)}$

Question 8

Root Locus Analysis and Gain Selection, Stability, Second Order Model, Step Response Specifications.

A unit feedback control system is to be stabilized using a Proportional Controller, as shown in Figure Q8.1.

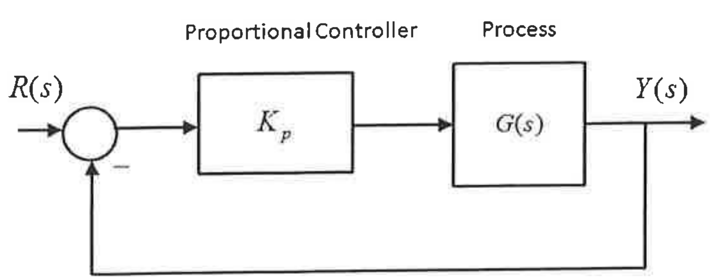


Figure Q8.1

The process transfer function is described as follows:

$$G(s) = \frac{5(s+3)}{(s+2)(s+5)(s+10)}$$

1) **(10 marks)** Sketch the Root Locus for the system, in the space provided in Figure Q8.2. Calculate all relevant coordinates: asymptotic angles, break-in/away points, the location of the centroid and the coordinates of the crossover with the Imaginary axis, i.e. ω_{osc} and the corresponding value of the critical gain, K_{crit}, at which the system becomes marginally stable.

2) **(7 marks)** It is required that the unit step response of the Closed Loop system exhibits Percent Overshoot of approximately 5%. Determine the corresponding Proportional Gain value, K_{op}, and calculate estimates of the following specs: Settling Time, $T_{settle(\pm 5\%)}$, Rise Time, $T_{rise(0-100\%)}$, and Steady State Error, $e_{ss(step\%)}$.

3) **(3 marks)** Finally, briefly comment on any possible differences between the expected system response (i.e. of the dominant poles model) and the actual system response.

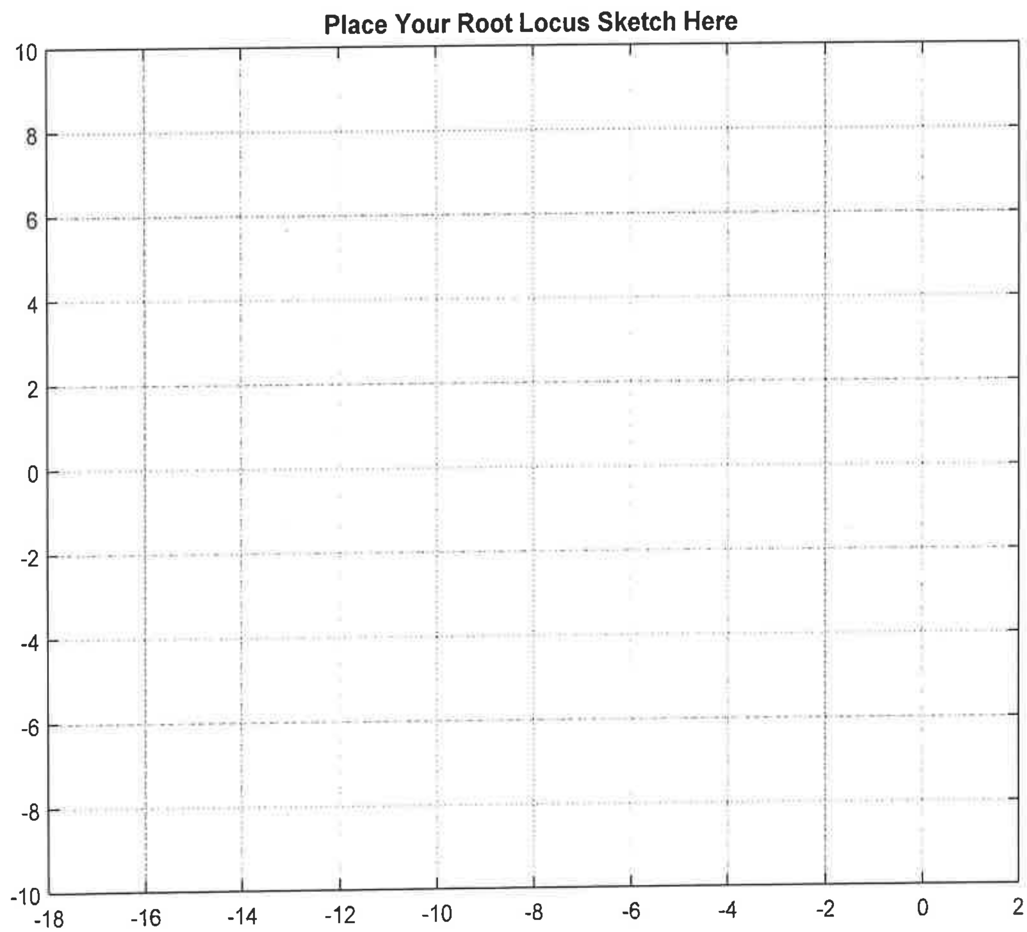


Figure Q8.2 – Root Locus of the System in Question 8