National Examinations December 2019

16-Elec-B3, Digital Communication Systems

3 Hours Duration

Notes:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.
2. This is a closed book exam. An approved Casio or Sharp calculator is permitted.
3. There are **5 questions** on this exam. **Any 4 questions constitute a complete paper.** Only the first 4 questions as they appear in your answer book will be marked.
4. Marks allocated to each question are noted in the left margin. A complete paper is worth 100 marks.

(25 marks) **Question 1.** This question concerns error-control coding.

(5 marks) a. Suppose a convolutional encoder has generator polynomials

$$g_1(D) = 1 + D^2$$
$$g_2(D) = 1 + D + D^2$$

For each input, the outputs are read out as g_1 first, then g_2. If the input to the convolutional encoder is 11010, the initial state is all-zero, and the encoder uses zero padding, give the encoded output.

(20 marks) b. For the same convolutional encoder, suppose the receiver observes 00111110000111. Assuming the encoder starts and ends in the all-zero state, use the Viterbi algorithm to determine the most likely input to the encoder, correcting any errors.

(25 marks) **Question 2.** This question concerns signal modulation and detection.

(5 marks) a. Consider signals $s_0(t)$ and $s_1(t)$, which are used to modulate the binary symbols "0" and "1", respectively, where

$$s_1(t) = \begin{cases} \sin(2\pi t/T), & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

and $s_0(t) = 0$. Sketch the two signals, and sketch the impulse response of the matched filter $m(t)$, assuming the filter is matched to $s_1(t)$, and assuming the filter output is sampled at time T.

(5 marks) b. In the absence of noise, what is the matched filter output at time T, if $s_1(t)$ is sent? Trig identity if you need it: $\sin^2 x = (1 - \cos 2x)/2$

(5 marks) c. At the sampling instant (time T), the matched filter output is corrupted by additive Gaussian noise with zero mean and variance σ^2. Give the optimal decision rule assuming that 0 and 1 are equiprobable.

(10 marks) d. Given that

$$\frac{1}{2}\operatorname{erfc}\left(\frac{t-\mu}{\sqrt{2\sigma^2}}\right) = \int_t^\infty \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) dx$$

and given your decision rule from part c, express the probability of error *given than a 0 was sent* in terms of erfc.

(25 marks) **Question 3.** This question concerns link budgeting.

(10 marks)

a. Consider a wireless system with transmitter power of 8 W, antenna gains of 3 dB, receiver losses of 6 dB, receiver noise figure of -174 dBm/Hz, a bandwidth of 10 MHz, and a fading margin requirement of 3 dB. Aside from free-space losses, no other gains or losses are present other than path loss. If the receiver requires a signal-to-noise ratio of at least 3 dB, what is the maximum allowed path loss (in dB)?

(5 marks)

b. Using a path loss of $30 \log_{10}(4 \pi df/c)$, where d represents the distance from transmitter to receiver, f represents the carrier frequency, and c represents the speed of light ($c = 3.0 \times 10^8$ m/s), and assuming a carrier frequency of 5 GHz, find the maximum distance so that the path loss criterion in part a is satisfied.

(5 marks)

c. From the path loss expression in part b, what is the path loss exponent?

(5 marks)

d. Suppose you receive a signal with power -30 dBm. Express this power level in watts.

(25 marks) **Question 4.** This question concerns the use of spread spectrum modulation.

(5 marks)

a. Explain the operation of direct sequence spread spectrum, including signal modulation and detection. In what sense is this technique "spread spectrum"?

(5 marks)

b. Explain the operation of frequency hopping spread spectrum, including signal modulation and detection. In what sense is this technique "spread spectrum"?

(5 marks)

c. For a system with bursty (highly irregular) traffic, is spread spectrum more appropriate than TDMA/FDMA? Briefly explain.

(5 marks)

d. Spread spectrum is often used for applications in the ISM band (e.g., Bluetooth). Why?

(5 marks)

e. Explain how spread spectrum trades off number of simultaneous users against interference.

(25 marks) **Question 5.** This question concerns sampling and D/A conversion.

a. NTSC-quality video has a bandwidth of 5 MHz. Using the Nyquist sampling criterion, what is the minimum sampling frequency in order to reconstruct the signal exactly? (5 marks)

b. Briefly explain pulse code modulation (PCM). If PCM is used to encode the signal from part a with 16 bits per sample, what is the required data rate to represent the signal? (If you didn't get an answer for part a, assume a value.) (5 marks)

c. A sinusoidal signal of frequency 600 Hz is sampled at a sampling frequency of 1000 Hz. Under aliasing, this signal would be indistinguishable from a sinusoidal signal of what frequency? (5 marks)

d. Suppose 8-bit PCM is used to sample a signal restricted between -2 V and +2 V. What is the maximum quantization error? (5 marks)

e. The data rate of MPEG-quality video is much less than your answer from part b. Give one reason why. (5 marks)