National Exams – May 2017

16–Mec-A6 Advanced Fluid Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question the candidate is urged to submit with the answer paper a clear statement of the assumptions made.
2. Candidates may use any approved Sharp/Casio calculator.
   The exam is OPEN BOOK.
3. Any FIVE (5) out of the 6 questions constitute a complete exam paper for a total of 100 MARKS.
   The first five questions as they appear in the answer book will be marked.
4. Each question is of equal value (20 marks) and question items are marked as indicated.
5. Clarity and organization of the answer are important.

(20) **Question 1**

A pressurized air canister has an internal volume of 0.5 m^3. The air (γ=1.4, R=287 J/(kg K), C_p=1,004.5 J/(kg K)) inside the canister is kept at a constant temperature of 300 K. Initially, the air inside the canister is pressurized to 10.0 MPa. The canister outlet consists of a valve shaped like a convergent-divergent nozzle. The throat area is given as A_T=1 mm^2 and the exit area as A_E=3.5 mm^2. The valve is opened to allow air to flow through. The air exhaust to atmosphere (P_b=100 kPa). It can be assumed that frictional losses are negligible. It can also be assumed that the air is still inside the canister (before entering the valve section).

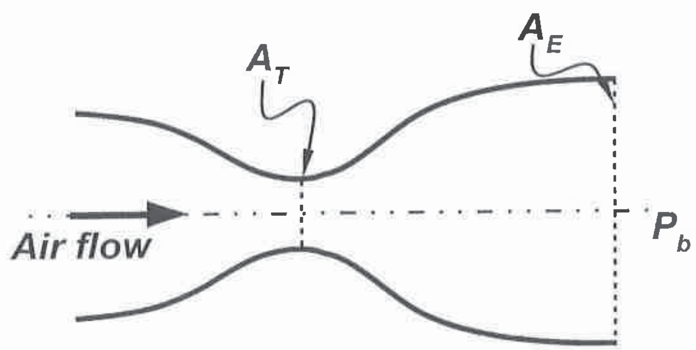


Figure 1: Cross-sectional schematic of valve section.

(5) (a) Determine the flow rate, exit temperature and speed of the air immediately after opening the valve (i.e. when the internal pressure is 10.0 MPa).

(5) (b) Determine the flow rate, exit temperature and speed of the flow when the internal pressure of the canister reaches 5.0 MPa.

(5) (c) At what internal pressure does a shock form exactly at the exit of the convergent-divergent nozzle? What is the flow rate at this point? What is the air temperature and speed immediately after the exit?

(5) (d) Below what canister pressure is the valve (nozzle) no longer choked?

(20) **Question 2**

A long municipal discharge pipe can be modelled as a source of strength m. The pipe is placed at a distance b from the bottom of a deep containment tank as shown in Figure 2. The city engineers want to drain the tank, but do not want to interrupt the discharge during the operation. The drain can be modelled as a sink of strength $-2m$ placed at the centre of the tank directly below the pipe, as shown in the figure. The city supervisor is concerned about the forces generated on the pipe. Your consulting company is asked to estimate these forces using potential flow theory.

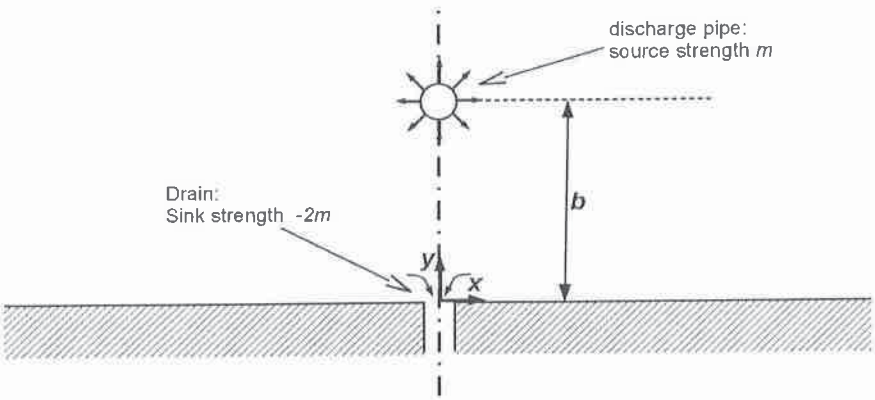


Figure 2: Cross-sectional view of discharge pipe next to a tank bed with a drain.

Assuming that the waste water density is the same as the water in the discharge tank, that the tank bed is flat, that the fluid velocity far from the discharge is negligible, and that the free surface effects can be neglected, determine:

(5) (a) The stream function that will represent this flow.

(5) (b) Verify that the tank bed is correctly simulated.

(5) (c) The velocity distribution along the tank bed.

(5) (d) The forces acting on the discharge pipe.

(20) **Question 3**

Water flows at a steady rate of Q=15 l/s through a flanged curved pipe. The water enters horizontally and exits at an angle of α=30° relative to the horizontal plane. The inlet and exit internal pipe diameters are D=10 cm and d=5 cm, respectively.

The velocity profiles in the pipe and at the exit may be assumed uniform.

At the exit, a free jet condition may be assumed.

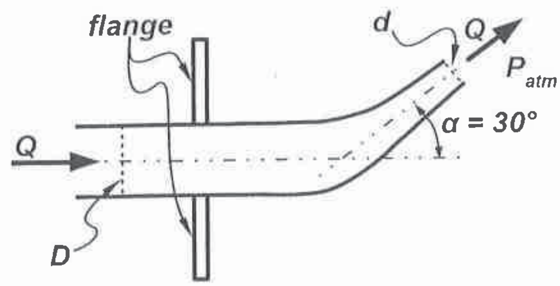


Figure 3: Schematic of curved pipe with flange.

(20) (a) Neglecting frictional losses and gravitational effects, estimate the forces (magnitude and direction, or F_x and F_y) acting on the flange.

(20) **Question 4**

The lubrication of bearings is a problem in fluid mechanics. The oil or lubricant between the bearing and the slider is a viscous fluid and most bearings operate in the laminar range with a very small Reynolds number. The spacing between the bearing and the slider (the flow gap) is much smaller than the length of the slider so that the flow becomes fully developed throughout most of the gap. Because the Reynolds number is so small, the inertia of the fluid is negligible compared with the pressure and viscous forces.

Consider a step bearing moving at velocity U, as shown in Figure 4. The spacings h_1 and h_2 are much less than L_1 and L_2 and the width of the bearing in the z direction is assumed to be very large, so that leakage in the z direction can be neglected. To facilitate the analysis, we attach coordinate systems to the slider with the origins at the beginning of each bearing section and we invert the direction of y, such that the relative velocity is $u = 0$ when $y = 0$, and $u = -U$ when $y = h$ in the respective sections.

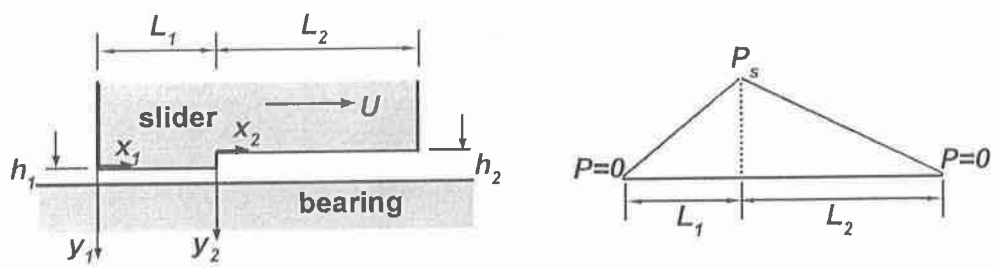


Figure 4: The step bearing and its pressure distribution.

(5) (a) Assuming incompressible flow of a Newtonian fluid, simplify the equations of motion (the Navier-Stokes equations) in x and in y directions.

(5) (b) Find an expression for the total flow rate and for the pressure change between any two points in a single section of the bearing.

(5) (c) Find an expression for the relative pressure distribution in the step bearing, assuming a zero gauge pressure at the inlet and outlet, as shown in Figure 4.

(5) (d) Calculate the total load carrying capacity per unit length of z for a step bearing with the following conditions:
U=0.5 m/s ; μ_{oil}=3.85 (N s)/m^2; ρ_{oil}=900 kg/m^3
h_1=1 mm ; L_1=30 mm
h_2=2 mm ; L_2=100 mm

(20) **Question 5**

At a sudden contraction in a pipe the diameter changes from D_1 to D_2. The pressure drop, Δp, which develops across the contraction is a function of D_1 and D_2, as well as the velocity U in the larger pipe, and the fluid density ρ and viscosity μ.

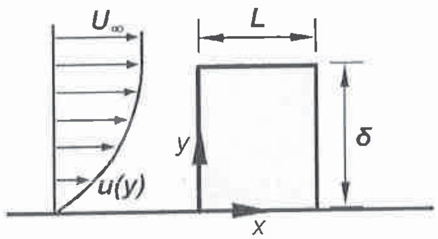
(15) (a) Using the Buckingham Pi theorem, and using D_1, U, and μ as repeating variables, determine a suitable set of dimensionless parameters.

(5) (b) Explain why would it be incorrect to include the velocity in the smaller pipe as an additional variable?

(20) **Question 6**

A thin, flat, two-sided plate of length L and height δ is attached orthogonally to a wall and oriented parallel to an approaching boundary layer flow, i.e. the plate is parallel to the x, y-plane. Assume that the boundary layer flow is fully turbulent and that the velocity profile follows a one-seventh power law:

$$u(y) = U_\infty \left(\frac{y}{\delta}\right)^{1/7}$$



(15) (a) Derive a formula for the drag coefficient of this plate.

(5) (b) How does this drag compare against the drag on the same plate immersed on a uniform stream with speed U_∞?