National Exams

07-Elec-B1, Digital Signal Processing

December 2013

3 Hours Duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Approved calculator is permitted. This is a CLOSED BOOK EXAM, but one aid sheet is allowed written on both sides.
3. There are five questions, however, FOUR(4) questions constitute a complete paper. The first four questions as they appear in the answer book will be marked.
4. All questions are of equal value.
5. Clarity and organization of the answer are important.

1. **(25 marks total)** Consider an LTI system defined by the difference equation

$$y[n] = -x[n] + 2x[n-1] - x[n-2].$$

(a) **(5 marks)** Determine the impulse response of the system.

(b) **(7 marks)** Determine the frequency response of this system. Express your answer in the form

$$H(e^{j\omega}) = A(e^{j\omega})e^{-j\omega n_d},$$

where $A(e^{j\omega})$ is a real function of ω. Explicitly specify $A(e^{j\omega})$ and the delay n_d of this system.

(c) **(8 marks)** Sketch a plot of the magnitude $|H(e^{j\omega})|$ and a plot of phase $\angle H(e^{j\omega})$.

(d) **(5 marks)** Suppose that the input to the system is

$$x_1[n] = 1 + e^{j0.5\pi n} \qquad -\infty < n < \infty$$

Use the frequency response function to determine the corresponding output $y_1[n]$.

2. **(25 marks total)** Consider the three sequences

$$v[n] = u[n] - u[n-6],$$

$$w[n] = \delta[n-2] + 2\delta[n-4] + \delta[n-6],$$

$$p[n] = v[n] * w[n].$$

   (a) **(10 marks)** Find and sketch the sequence $p[n]$.

   (b) **(10 marks)** Find and sketch the sequence $r[n]$ such that $r[n] * v[n] = \Sigma_{k=-\infty}^{n-1} p[k]$.

   (c) **(5 marks)** Is $p[-n] = v[-n] * w[-n]$? Justify your answer.

3. **(25 marks total)** A causal LTI system has system function

$$H(z) = \frac{1 - z^{-1}}{(1 - 0.6z^{-1})(1 + 0.6z^{-1})}$$

(a) **(9 marks)** Determine the output of the system when the input is $x[n] = u[n]$.

(b) **(8 marks)** Determine the input $x[n]$ so that the corresponding output of the above system is $y[n] = \delta[n] - \delta[n-1]$.

(c) **(8 marks)** Determine the output $y[n]$ when the input is $x[n] = \cos(\frac{\pi}{3}n)$ for $-\infty < n < \infty$. You may leave your answer in any convenient form.

4. **(25 marks total)** Figure 1. shows a continuous-time filter that is implemented using an LTI discrete-time filter with frequency response $H(e^{j\omega})$ as depicted in Figure 2. Note that Ω denotes continuous-time frequency and ω denotes discrete-time frequency. If the continuous-time Fourier transform of $x_c(t)$, namely $X_c(j\Omega)$, is as shown in Figure 3, with $\Omega = \pi \times 10^5$ and $\omega_c = \frac{\pi}{5}$, sketch and label $X(e^{j\omega})$, $Y(e^{j\omega})$ and $Y_c(j\Omega)$ for each of the following cases.

   (a) **(9 marks)** $\frac{1}{T_1} = \frac{1}{T_2} = 2 \times 10^5$

   (b) **(8 marks)** $\frac{1}{T_1} = 4 \times 10^5, \frac{1}{T_2} = 10^5$

   (c) **(8 marks)** $\frac{1}{T_1} = 10^5, \frac{1}{T_2} = 3 \times 10^5$

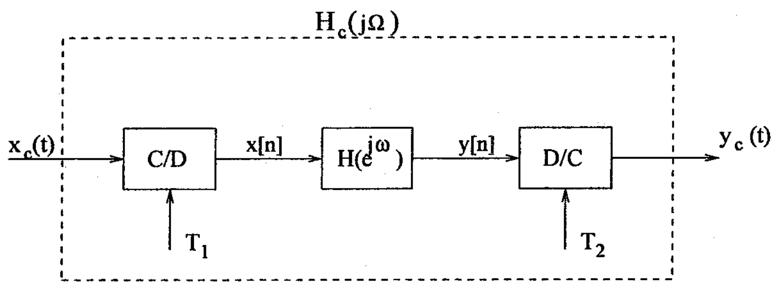

Figure 1:

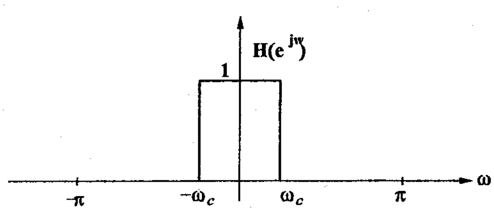

Figure 2:

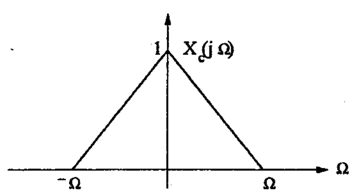

Figure 3:

5. **(25 marks total)** Consider the real finite-length sequence $x[n]$ shown in Figure 4.

   (a) **(8 marks)** Sketch the finite-length sequence $y[n]$ whose six-point DFT is

   $$Y[k] = W_6^{5k} X[k]$$

   where $X[k]$ is the six-point DFT of $x[n]$.

   (b) **(7 marks)** Sketch the finite-length sequence $w[n]$ whose six-point DFT is

   $$W[k] = Im\{X[k]\}$$

   (note: *Im* stands for the imaginary part)

   (c) **(10 marks)** Sketch the finite-length sequence $q[n]$ whose three-point DFT is

   $$Q[k] = X[2k+1], \qquad k = 0, 1, 2.$$

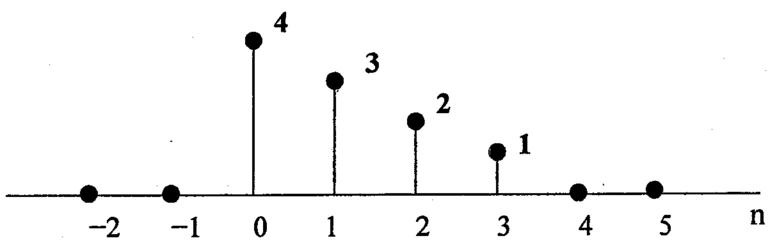

Figure 4: