May 2013

NATIONAL EXAMS

Phys-A6: Solid State Physics

3 hours duration

NOTES:

1. If doubt exits as to the interpretation of any question, the candidate must submit with the answer paper, a clear statement of any assumption made.

2. Candidates may use one of two calculators, the Casio or Sharp approved models.

3. This is a CLOSED BOOK EXAM.
   Useful constants and equations have been annexed to the exam paper.

4. **Any FIVE (5) of the SEVEN (7)** questions constitute a complete exam paper.
   The first five questions as they appear in the answer book will be marked.

5. When answering questions, candidates must clearly indicate units for all parameters used or computed.

MARKING SCHEME

Questions	*Marks*			
1	(a) 5	(b) 5	(c) 10	
2	(a) 6	(b) 6	(c) i. 4	(c) ii. 4
3	(a) 10	(b) 7	(c) 3	
4	(a) 9	(b) 4	(c) 7	
5	(a) 5	(b) 10	(c) 5	
6	(a) 8	(b) 12		
7	(a) 8	(b) 8	(c) 4	

1. A face centered cubic (*fcc*) lattice and its primitive cell are show in Figure P1a and a lattice plane of this crystal is shown in Figure P1b.

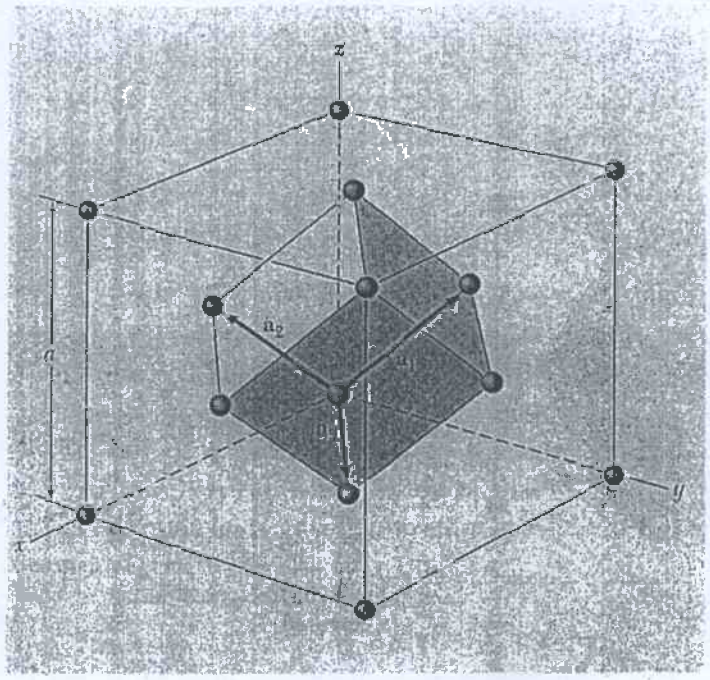

Figure P1a

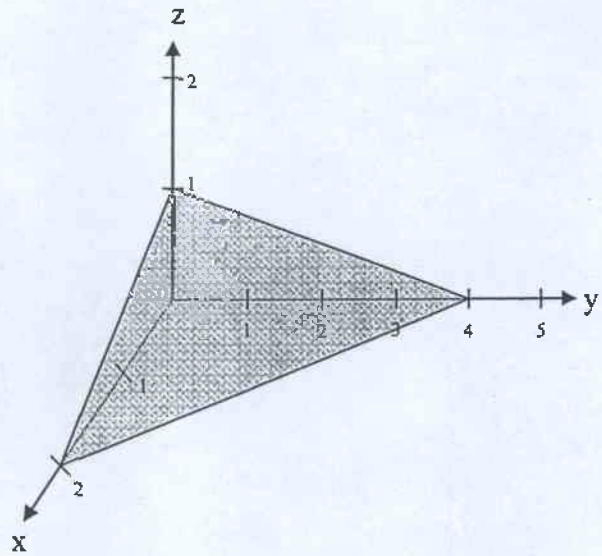

Figure P1b

5 pts (a) Calculate the *packing fraction* for this cubic lattice. [Note: $V_{sphere} = \frac{4\pi r^3}{3}$]

5 pts (b) Find the Miller indices for the plane shown in Figure P1b.

10 pts (c) Find the primitive translation vectors $\mathbf{b}_1$, $\mathbf{b}_2$ and $\mathbf{b}_3$ for the *reciprocal lattice* for the *fcc* lattice.

2. The interaction between two inert gas atoms takes the form of the normalized potential $U(R)/\epsilon$ shown in Figure P2.

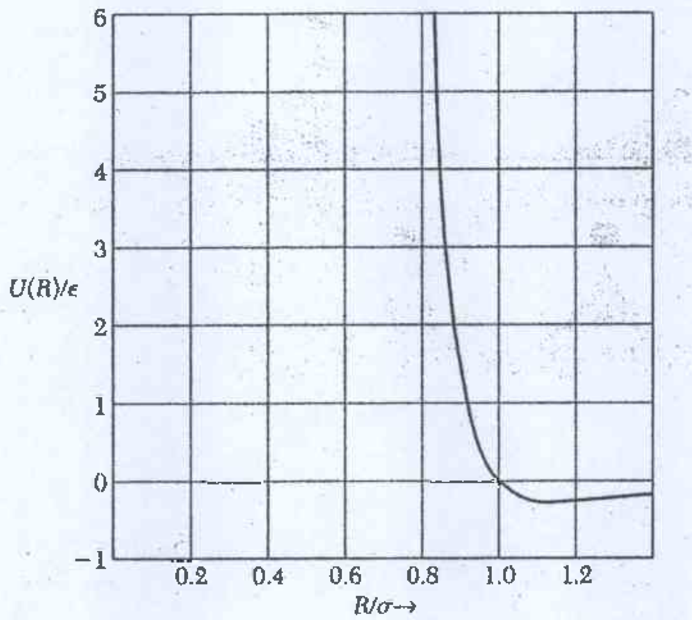

Figure P2

6 pts (a) What is the name of this potential?

6 pts (b) Briefly explain what the negative part of the curve signifies?

(c) Measurements done at very low temperature for hydrogen (H_2) gave the following parameter values

$\epsilon = 40\times10^{-16}$ erg $\quad\quad \sigma = 3$ Å

Assuming that at this temperature the H_2 molecules are hard spheres in an fcc lattice structure:

4 pts i. Calculate the value of the interaction potential (in Joules) between H_2 molecules when they are 3.6 Å apart.

4 pts ii. Calculate the cohesive energy in kJ per mole of H_2.

3. Consider vibrations in a crystal with a monatomic basis where each atom has a mass M and a force constant C between nearest-neighbour lattice planes. Plane displacements are illustrated in Figure P3.

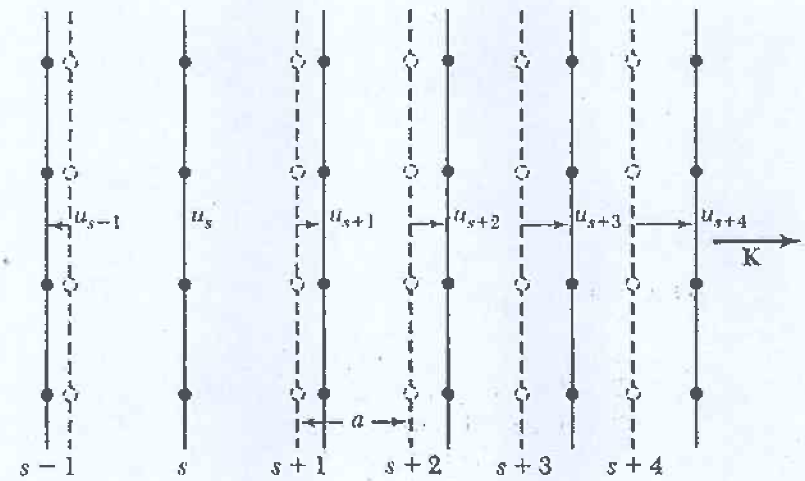

Figure P3

Assuming displacements of the form $u_s = u\ exp(isKa)$ all having the time dependence $exp(-i\omega t)$ and considering only nearest planes,

10 pts (a) Show that the dispersion relation $\omega(K)$ is given by $\omega = \sqrt{\left(\frac{4C}{M}\right)}\left|\sin\left(\frac{Ka}{2}\right)\right|$

7 pts (b) Plot the dispersion relation for the *first Brillouin zone.*

3 pts (c) What sort of waves are present at the the *first Brillouin zone* boundaries?

4. Particles which behavior follows the Fermi-Dirac distribution are called *fermions*. The 3-dimensional Fermi surface of fermions is shown in Figure P4. Just like free electrons, the helium-3 (He^3) atoms behave like fermions. He^3 is composed of 3 atomic mass units: 2 protons and 1 neutron. The density of He^3 near absolute zero temperature is 0.081 g/cm^3.

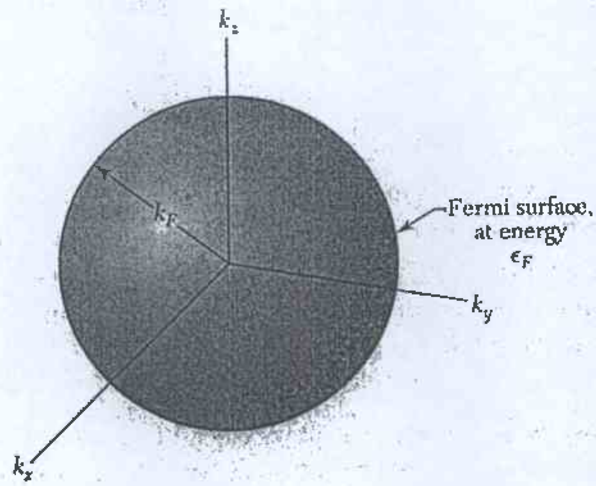

Figure P4

9 pts (a) Show that He^3 atoms have a Fermi energy ϵ_F of about 7 x 10^{-16} erg.

4 pts (b) What is the Fermi temperature T_F that corresponds to this Fermi energy?

7 pts (c) Assuming that the *chemical potential* is approximately equal to ϵ_F, what is the probability that an atom of He^3 would occupy an energy level of $\epsilon = 20\epsilon_F$ at T= 45 ^oK ?

5. Electron occupancy of various allowed energy bands for five cases is shown in Figure P5. The grey areas indicate states filled with electrons.

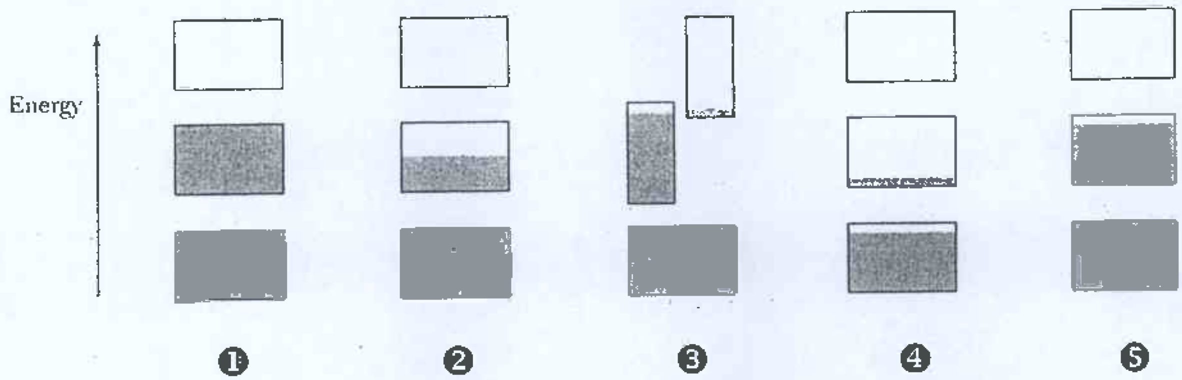

Figure P5

5 pts (a) For each of the five cases shown in Figure P5, specify whether the type of crystal is an insulator, a metal, a semiconductor, or a semimetal.

10 pts (b) Assuming that intrinsic silicon (Si) has a constant *band gap* of 1.08 eV, calculate the electron concentration of intrinsic silicon (Si) at room temperature (T=300 °K) if measurements on this semiconductor have shown that the effective mass of an electron is $1.1m$ and the effective mass of a hole is $0.56m$ where m is the mass of an electron at rest.

5 pts (c) The Fermi level of an intrinsic semiconductor is situated in the *band gap*, half way between the valence and conduction bands. If this semiconductor is now heavily doped with *donors*, briefly explain what happens to the position of the Fermi level?

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6. The following questions refer to magnetism present or induced in crystal lattices.

8 pts (a) Briefly explain how a *diamagnetic* material differs from a *paramagnetic* material.

12 pts (b) Calculate the magnetic susceptibility of a diamagnetic material having the following properties:

density: 1.785×10^{-4} g/cm^3
atomic radius: 3.1×10^{-9} cm
number of neutrons: 2
number of protons: 2
number electrons: 2

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7. The diffusion constants D_0 and activation energies **E** for various lattice defects such as impurity atoms or vacancies are listed below.

Host crystal	Atom	D_0 (cm^2/s)	E (eV)
Cu	Cu	0.20	2.04
Cu	Zn	0.34	1.98
Ag	Ag	J0.40	1.91
Ag	Cu	1.2	2.00
Ag	Au	0.26	1.98
Ag	Pb	0.22	1.65
Na	Na	0.24	0.45
U	U	0.002	1.20

Host crystal	Atom	D_0 (cm^2/s)	E (eV)
Si	Al	8.0	3.47
Si	Ga	3.6	3.51
Si	In	16.0	3.90
Si	As	0.32	3.56
Si	Sb	5.6	3.94
Si	Li	0.002	0.66
Si	Au	0.001	1.13
Ge	Ge	10.0	3.1

8 pts (a) Briefly explain how the presence of impurities such as arsenic (As) in pure silicon (Si) can actually be useful in solid state devices.

8 pts (b) Sodium (Na) has a density of 0.971 g/cm^3 and its atomic weight is 23 amu. If the energy to take an atom of Na from its normal lattice site to a lattice site at the surface of the crystal is 1.05 eV, calculate the concentration of defect vacancies present at a temperature of 300 °K.

4 pts (c) Determine at what rate aluminum (Al) atoms would diffuse into a lattice of pure silicon (Si) at a temperature of 1200 °K.

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USEFUL EQUATIONS AND CONSTANTS

(1) $sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ $\qquad cos\theta = \frac{1}{2}[\exp(i\theta) + \exp(-i\theta)]$

(2) $\boldsymbol{T} = u_1\boldsymbol{a}_1 + u_2\boldsymbol{a}_2 + u_3\boldsymbol{a}_3$

(3) $\boldsymbol{G} = v_1\boldsymbol{b}_1 + v_2\boldsymbol{b}_2 + v_3\boldsymbol{b}_3$

(4) $\boldsymbol{p} = \boldsymbol{r} \times \boldsymbol{t} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}(\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z}) = \begin{pmatrix} r_2t_3 - r_3t_2 \\ r_3t_1 - r_1t_3 \\ r_1t_2 - r_2t_1 \end{pmatrix}(\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z})$ where $\boldsymbol{r} = r_1\boldsymbol{x} + r_2\boldsymbol{y} + r_3\boldsymbol{z}$, $\boldsymbol{t} = t_1\boldsymbol{x} + t_2\boldsymbol{y} + t_3\boldsymbol{z}$

(5) $V_{min} = |\boldsymbol{a}_1 \cdot \boldsymbol{a}_2 \times \boldsymbol{a}_3|$

(6) $\boldsymbol{b}_1 = 2\pi\frac{\boldsymbol{a}_2\times\boldsymbol{a}_3}{\boldsymbol{a}_1\cdot\boldsymbol{a}_2\times\boldsymbol{a}_3}$ $\qquad \boldsymbol{b}_2 = 2\pi\frac{\boldsymbol{a}_3\times\boldsymbol{a}_1}{\boldsymbol{a}_1\cdot\boldsymbol{a}_2\times\boldsymbol{a}_3}$ $\qquad \boldsymbol{b}_3 = 2\pi\frac{\boldsymbol{a}_1\times\boldsymbol{a}_2}{\boldsymbol{a}_1\cdot\boldsymbol{a}_2\times\boldsymbol{a}_3}$

(7) $2d\sin\theta = n\lambda$ $\qquad \Delta\boldsymbol{k} = \boldsymbol{G}$ $\qquad 2\boldsymbol{k}\cdot\boldsymbol{G} = G^2$

(8) $U(R) = 4\epsilon\left[\left(\frac{\sigma}{R}\right)^{12} - \left(\frac{\sigma}{R}\right)^{6}\right]$

(9) $U_{tot} = -(2.15)(4N\varepsilon)$

(10) $F_s = C(u_{s+1} - u_s) - C(u_{s-1} - u_s)$

(11) $M\frac{d^2u_s}{dt^2} = C(u_{s+1} - u_s) - C(u_{s-1} - u_s)$

(12) $f(\in) = \frac{1}{exp\left[\frac{\in-\mu}{k_BT}\right]+1}$

(13) $\in_F = \frac{\hbar}{2m}\left(\frac{3\pi^2N}{V}\right)^{2/3}$

(14) $np = 4\left(\frac{k_BT}{2\pi\hbar^2}\right)^3 (m_em_h)^{3/2}\exp\left(\frac{-E_g}{k_BT}\right)$

(15) $n_i = p_i = 2\left(\frac{k_BT}{2\pi\hbar^2}\right)^{3/2} (m_em_h)^{3/4}\exp\left(\frac{-E_g}{2k_BT}\right)$

(16) $\mu = \frac{E_g}{2} + \frac{3}{4}k_BT\ \ln(m_h/m_e)$

(17) $\chi = -\frac{\mu_oNZe^2}{6m}\langle r^2\rangle$

(18) $\frac{n}{N-n} = exp\left(\frac{-E_V}{k_BT}\right)$

(19) $D = D_oexp\left(\frac{-E}{k_BT}\right)$

Quantity	Symbol	Value	CGS	SI
Velocity of light	c	2.997925	10^{10} cm s^{-1}	10^{8} m s^{-1}
Proton charge	e	1.60219	—	10^{-19} C
		4.80325	10^{-10} esu	—
Planck's constant	h	6.62620	10^{-27} erg s	10^{-34} J s
	$\hbar = h/2\pi$	1.05459	10^{-27} erg s	10^{-34} J s
Avogadro's number	N	6.02217×10^{23} mol^{-1}	—	—
Atomic mass unit	amu	1.66053	10^{-24} g	10^{-27} kg
Electron rest mass	m	9.10956	10^{-28} g	10^{-31} kg
Proton rest mass	M_p	1.67261	10^{-24} g	10^{-27} kg
Proton mass/electron mass	M_p/m	1836.1	—	—
Reciprocal fine structure constant $\hbar c/e^2$	$1/\alpha$	137.036	—	—
Electron radius e^2/mc^2	r_e	2.81794	10^{-13} cm	10^{-15} m
Electron Compton wavelength $\hbar/mc$	λ_e	3.86159	10^{-11} cm	10^{-13} m
Bohr radius $\hbar^2/me^2$	r_0	5.29177	10^{-9} cm	10^{-11} m
Bohr magneton $e\hbar/2mc$	μ_B	9.27410	10^{-21} erg G^{-1}	10^{-24} J T^{-1}
Rydberg constant $me^4/2\hbar^2$	R_∞ or Ry	2.17991	10^{-11} erg	10^{-18} J
		13.6058 eV		
1 electron volt	eV	1.60219	10^{-12} erg	10^{-19} J
	eV/h	2.41797×10^{14} Hz	—	—
	eV/hc	8.06546	10^{3} cm^{-1}	10^{5} m^{-1}
	eV/k_B	1.16048×10^{4} K	—	—
Boltzmann constant	k_B	1.38062	10^{-16} erg K^{-1}	10^{-23} J K^{-1}
Permittivity of free space	ϵ_0	—	1	$10^7/4\pi c^2$
Permeability of free space	μ_0	—	1	$4\pi \times 10^{-7}$ N/A^2