National Exams May 2019

16-Mec-B12, Robot Mechanics

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit, with the answer paper, a clear statement of any assumptions made.
2. This is a CLOSED BOOK EXAM. Candidates are allowed to bring ONE aid sheet 8.5" X 11" hand-written on both sides containing notes and formulae. A Casio or Sharp approved calculator is permitted.
3. FIVE (5) questions constitute a complete exam paper. There are two sections in this exam. There are five questions in the first section. The candidate will need to answer **ONLY three**. In the second section there are two questions which the candidate **MUST answer both**.
4. Each question is of equal value.
5. Question value in marks is shown in parentheses at the end of each question part.
6. Logical order, clarity, and organization of the solution steps are important.

Nomenclature:

The unit vectors of coordinate system *A* are denoted by $\hat{X}_A, \hat{Y}_A, \hat{Z}_A$.

The leading superscript such as *A* in ${}^{A}V$ indicates the coordinate system to which the vector *V* is referenced.

The leading subscript and superscript such as *A* and *B* in ${}^{B}_{A}T$ indicate the transformation of coordinate frame *A* relative to *B* by matrix *T*.

Section 1: Answer ONLY three out of five

(20)

1. A position vector is given by ${}^{A}P = \begin{bmatrix} -5 & 3 & 4 \end{bmatrix}^T$ and a velocity vector by ${}^{B}V = \begin{bmatrix} 10 & 20 & -15 \end{bmatrix}^T$. Given the Homogeneous Transformation

$$ {}^{A}_{B}T = \begin{bmatrix} \sqrt{3}/2 & -0.5 & 0 & 11 \\ 0.5 & \sqrt{3}/2 & 0 & -3 \\ 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix}; $$

Compute:

a) ${}^{B}P$ **(12)**

b) ${}^{A}V$ **(8)**

(20)

2. The following frame definitions are given

$$ {}^{U}_{A}T = \begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad {}^{B}_{A}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -0.5 & 10 \\ 0 & 0.5 & \sqrt{3}/2 & -20 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ {}^{C}_{U}T = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 & -3 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

a) Draw a frame diagram to show their arrangement graphically. **(5)**

b) Solve for ${}^{B}_{C}T$. **(15)**

(20)

3. Solve for the coefficients of two cubics that are connected in a two-segment spline with continuous acceleration at the intermediate via point. The initial angle is $\theta_0 = -20°$, the via point is $\theta_v = 45°$, and the goal point is $\theta_g = 25°$. The first cubic is:

$$\theta(t) = a_{10} + a_{11}t + a_{12}t^2 + a_{13}t^3$$

and the second is:

$$\theta(t) = a_{20} + a_{21}t + a_{22}t^2 + a_{23}t^3$$

Each cubic will be evaluated over an interval starting at $t_{i1} = 0, t_{i2} = 0$ and ending at $t_{f1} = 4s$ and $t_{f2} = 4s$.

(20)

4. A 4R manipulator is shown schematically in Figure 1. The nonzero link parameters are $\alpha_1 = -90°$, $d_2 = 1$, $\alpha_2 = 45°$, and $a_3 = 1$, and the mechanism is pictured in the configuration corresponding to $\Theta = [0, \ 0, \ 90°, \ 0]^T$. Each joint has ±180° as limits. Find all values of θ_3 such that ${}^{0}P_{4ORG} = [0.0, \ 1.0, \ 1.414]^T$

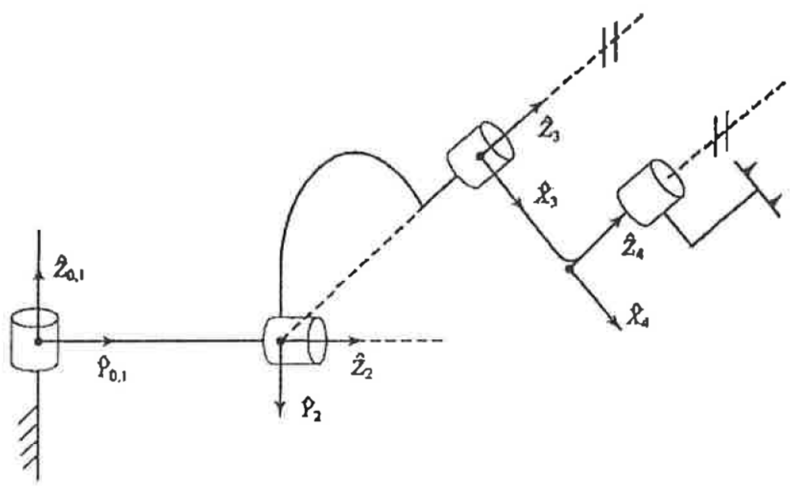

Figure 1

(20)

5. Derive the equations of motion for the *PR* manipulator shown in Figure 2. Neglect friction, but include gravity. (Here, $\hat{X}_o$ is upward.) The inertia

tensors of the links are diagonal, with moments I_{xx1}, I_{yy1}, I_{zz1} and I_{xx2}, I_{yy2}, I_{zz2}. The centers of mass for the links are given by

$$
{}^{1}P_{C1} = \begin{bmatrix} 0 \\ 0 \\ -l_1 \end{bmatrix}, \quad {}^{2}P_{C2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.
$$

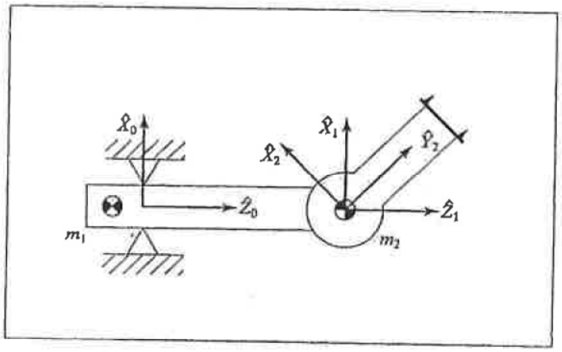

Figure 2

Section 2: Answer both questions

(20)

6. Consider the manipulator, shown in Figure 3 below, in its rest position with positive joint motion in the directions indicated.

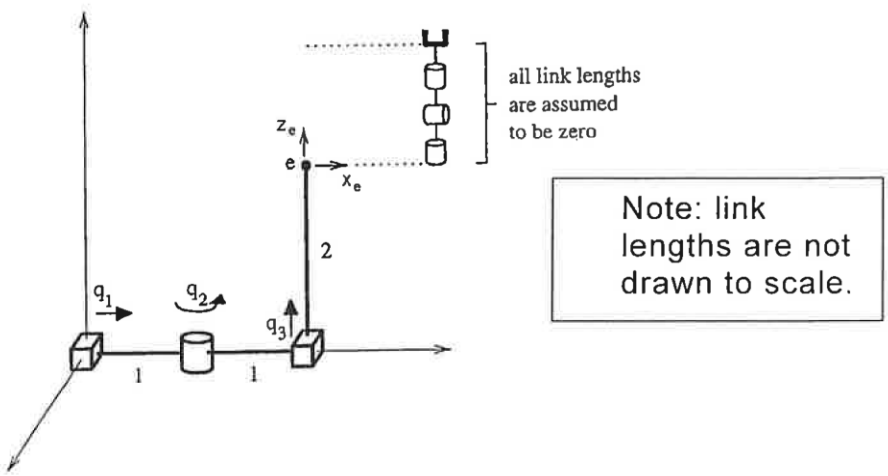

Figure 3

a) Using the axes provided for frames 0 and 3, show your axis assignments on a sketch. **(5)**
b) Derive the DH table for the manipulator without the wrist (i.e. to the end point *e*). **(5)**

c) Calculate ${}^{0}_{3}T$. **(10)**

(20)
7. For the manipulator, shown in Figure 3 question 6, without wrist:

a) Find the Jacobian. **(12)**

Assuming the following motion limits for the joints:

$$0 < q_1 < 1,\ 0 < q_2 < 360^\circ,\ 0 < q_3 < 1,$$

b) Sketch the reachable workspace of the manipulator without the wrist as projections on the x_0-y_0, y_0-z_0 and x_0-z_0 planes. **(6)**

c) What can you say about the dexterous workspace of the manipulator without the wrist? **(2)**