# Professional Engineers Ontario 

Exam<br>07-Elec-A6 Power Systems and Machines

December 2016

## Notes:

1. Do only five questions from a choice of six. FIVE (5) questions constitute a complete exam paper. Unless you indicate otherwise, the first five questions as they appear in the answer booklet will be the only ones marked. All questions are of equal value.
2. Neatness is important. Start each question on a new page, and clearly indicate the question number. Only work written on the right hand pages of the answer booklets will be marked. Use the pages on the left side for rough work only - work presented on the left hand side pages will NOT be marked.
3. You may use one of the approved Casio or Sharp calculators.
4. This is a closed book exam. Formula sheets are attached.
5. All ac voltages and currents are rms values unless noted otherwise. For three-phase circuits, all voltages are line-to-line voltages unless noted otherwise, and power is total real power unless noted otherwise.
6. You are strongly encouraged to use a pencil and eraser for this exam.

If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper a clear statement of any assumptions made.

## Question 1.

A plant fed by three-phase source $480 \mathrm{~V}, 60 \mathrm{~Hz}$ (Figure 1) is composed of 4 loads:
i. a three-phase induction motor with the following rated specifications: 250 kW at 0.9 lagging power factor;
ii. a second three-phase induction motor with the following rated specifications: 350 kW at 0.75 lagging power factor;
iii. a synchronous motor that draws a real power of 300 kW at 0.9 lagging power factor; and,
iv. three impedances, $\mathbf{Z}$, connected in $\Delta$, where $\mathbf{Z}=2.5 \angle 30^{\circ} \Omega$

Determine:
a) the total active and reactive power absorbed by the plant; and,
b) the total line current (magnitude and phase).

If the synchronous motor operates instead at 0.9 leading power factor, determine:
c) the new synchronous line current;
d) the new total line current and power factor; and,
e) the indicated reading of each wattmeter (W1 and W2).


Figure 1

## Question 2.

A single-phase transformer has the following nameplate ratings: $100 \mathrm{kVA}, 7200 / 240 \mathrm{~V}$ and 60 Hz . The following results are obtained from open-circuit and short-circuit tests:

| Test | Voltage | Current | Real Power Input |
| :---: | :---: | :---: | :---: |
| Open-circuit | 7200 V | 0.65 A | 425 W |
| Short-circuit | 250 V | 13.89 A | 1420 W |

a) Determine the parameters of this transformer, all referred to the primary side.
b) The secondary of this transformer is connected to a load that draws $100 \%$ of the rated power at rated voltage and at 0.9 power factor lagging. Determine:
(i) the efficiency of the transformer;
(ii) the voltage regulation; and,
(iii) the phasor diagram of the transformer at the primary (source) side.

## Question 3.

A 3-phase, $440 \mathrm{~V}, 60 \mathrm{~Hz}, 75 \mathrm{~kW}$, Y-connected, six-pole squirrel cage induction motor is operating at a slip of 0.04 . Core and rotational losses are evaluated at 1400 W and 1450 W , respectively. The motor parameters, in ohms/phase, and referred to the stator, are:

$$
\mathrm{R}_{1}=0.082 \Omega ; \mathrm{X}_{1}=0.19 \Omega ; \mathrm{R}_{2}^{\prime}=0.070 \Omega ; \mathrm{X}_{2}^{\prime}=0.18 \Omega ; \mathrm{X}_{\mathrm{m}}=7.2 \Omega
$$

Determine:
a. the stator armature current;
b. the rotor current;
c. stator input power;
d. stator copper loss;
e. rotor power input;
f. rotor power developed;
g. total output power in watts and horsepower;
h. motor efficiency; and,
j. output torque.

## Question 4.

A $30 \mathrm{hp}, 240 \mathrm{~V}, 1150 \mathrm{rpm}$ dc shunt motor operating at rated conditions has an efficiency of $88.5 \%$. The armature circuit resistance and field circuit resistance are $0.096 \Omega$ and $93.6 \Omega$, respectively. Calculate the following:
(a) the armature current;
(b) the output torque;
(c) the mechanical developed power;
(d) the mechanical developed torque; and,
(e) the external resistance required in series with the armature to limit the starting torque to $200 \%$ of rated torque.

If the load torque is now reduced to $40 \%$ rated torque, find:
(f) the new armature current, $\mathrm{I}_{\mathrm{a}}$;
(g) the new operating speed; and,
(h) the external resistance required in series with the armature to maintain rated speed.

## Question 5.

A 3-phase, 10MVA, $6600 \mathrm{~V}, 60 \mathrm{~Hz}$, 4-pole, Y-connected synchronous motor has a synchronous reactance of $10 \Omega$ per phase and negligible armature resistance. At no-load the motor draws an armature current of 10 A at a power factor of 0.05 leading.

For the no-load condition, (a) draw the phasor diagram, and calculate:
(b) the excitation voltage;
(c) the power angle, $\delta$; and,
(d) the real input power to the motor

If the field current and excitation remain unchanged, but the motor is loaded such that the power factor rises to unity, calculate:
(e) the new stator current;
(f) the new power angle, $\delta$;
(g) the new real input power to the motor; and,
(h) the efficiency of the motor.

## Question 6.

Find the current required to establish a flux of 2 mWb in the air gap of the magnetic structure shown in Figure 2 below (the magnetization curve is also provided). The coil has 500 turns, and the structure the following dimensions:

$$
\begin{array}{llll}
\mathrm{A}_{1}=40 \mathrm{~cm}^{2} & \mathrm{~A}_{2}=12 \mathrm{~cm}^{2} & \mathrm{~A}_{3}=\mathrm{A}_{4}=25 \mathrm{~cm}^{2} & \mathrm{~A}_{\mathrm{g}}=26 \mathrm{~cm}^{2} \\
l_{1}=40 \mathrm{~cm} & l_{2}=24 \mathrm{~cm} & l_{3}=l_{4}=26 \mathrm{~cm} & l_{g}=25 \times 10^{-3} \mathrm{~cm}
\end{array}
$$



Figure 2

END OF THE EXAM

## Potentially useful formulae

$$
\begin{aligned}
& P=V I \cos \theta=\frac{V_{R}{ }^{2}}{R}=I^{2} R=\operatorname{Re}\left[\mathbf{V I}{ }^{*}\right] \\
& Q=V I \sin \theta=\frac{V_{X}^{2}}{X}=I^{2} X=\operatorname{Im}\left[\mathbf{V I}{ }^{*}\right] \\
& \mathbf{S}=\mathbf{V I}^{*} \\
& |\mathbf{S}|=\sqrt{P^{2}+Q^{2}}=V I=I^{2} Z=\frac{V^{2}}{Z} \\
& \text { p.f. }=\cos \theta=\frac{R}{Z}=\frac{P}{S} \\
& P_{T}=\sqrt{3} V_{L} I_{L} \cos \theta=3 P_{P} \quad P_{P}=V_{P} I_{P} \cos \theta \\
& Q_{T}=\sqrt{3} V_{L} I_{L} \sin \theta=3 Q_{P} \quad Q_{P}=V_{P} I_{P} \sin \theta \\
& S_{T}=\sqrt{3} V_{L} I_{L} \\
& S_{P}=V_{P} I_{P} \\
& B=\frac{\Phi}{A}=\mu H=\mu \frac{\mathscr{F}}{l}=\mu \frac{N i}{l} \quad\left[\frac{W b}{m^{2}}=T\right] \\
& H=\frac{N I}{l}=\frac{B}{\mu}=\frac{\Phi / A}{\mu} \quad\left[\frac{A-t}{m}\right] \\
& \mathscr{F}=N i=\Phi \frac{l}{\mu A}=\Re \Phi \quad[A-t] \\
& \Re=\frac{l}{\mu A} \quad\left[\frac{A-t}{W b}\right] \\
& \mu_{0}=4 \pi \times 10^{-7} \frac{W b}{A-t-m} \quad \mu=\mu_{0} \mu_{r} \\
& P_{e}=K_{t} f^{2} B^{2}{ }_{\text {max }} V_{v o l} \quad P_{h}=K_{h} f B^{x}{ }_{\text {max }} V_{v o l} \\
& L=\frac{N^{2}}{\Re}
\end{aligned}
$$

$$
\begin{aligned}
& I_{L}=I_{f}+I_{a} \\
& V_{t}=E_{a}+I_{a} R_{a} \\
& E_{a}=K_{a} \Phi \omega \\
& T=K_{a} \Phi I_{a} \\
& P_{\text {input }}=V_{t} I_{L} \\
& P_{d e v}=E_{a} I_{a}=T_{d e v} \omega_{m} \\
& P_{\text {out }}=P_{d e v}-P_{r o t}=T_{o u t} \omega_{m} \\
& P_{\text {rot }}=\text { No load } P_{d e v} \\
& n_{s}=120 \frac{f}{p} \\
& s=\frac{n_{s}-n_{m}}{n_{s}} \\
& P_{\text {input }}=3 V_{1} I_{1} \cos \theta \\
& P_{\text {gap }}=P_{\text {input }}-3 I_{1}^{2} R_{1}=3 I_{2}^{\prime 2} \frac{R_{2}^{\prime}}{s}=T_{d e v} \omega_{s} \\
& 3 I_{2}^{\prime 2} R_{2}^{\prime}=s P_{g a p} \\
& P_{d e v}=P_{g a p}-3 I_{2}^{\prime 2} R_{2}^{\prime}=(1-s) P_{g a p} \\
& P_{\text {out }}=P_{\text {dev }}-P_{\text {rot }}=T_{\text {out }} \omega_{m} \\
& \mathbf{E}_{\mathbf{a}}=\mathbf{V}_{\mathbf{t}}+\mathbf{I}_{\mathbf{a}}\left(R_{a}+j X_{s}\right) \\
& P=\frac{3 V_{t} E_{a}}{X_{s}} \sin \delta
\end{aligned}
$$

