National Exams May 2015

07-Mec-A3, SYSTEM ANALYSIS AND CONTROL

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
2. Candidates may use a Casio **or** Sharp approved calculator. This is a **closed book** exam. No aids other than semi-log graph papers are permitted.
3. Any four questions constitute a complete paper. Only the first four (4) questions as they appear in your answer book will be marked.
4. All questions are of equal value.

Question 1:

Construct asymptotic Bode magnitude plots for the following transfer functions.

(a) $$\frac{4}{s+2}$$

(b) $$\frac{4}{(0.4s+1)(s+1)}$$

Question 2:

(a) Calculate the unit step response of

$$G(s) = \frac{1}{(s+2)^2(s+1)}$$

(b) Calculate the unit step response of the system

$$G(s) = \frac{54}{(2s+6)(s^2+3s+9)}$$

Question 3:

Let Fig. 1 model a temperature control system with plant transfer function $G(s) = 1/[(s + 1)(s + 5)]$.

(a) With P control $G_c = K_c$, what is the system type number, and what is the gain?

(b) For $G_c = K_c$, find K_c for a damping ratio 0.5 and the corresponding steady-state error for a unit step input.

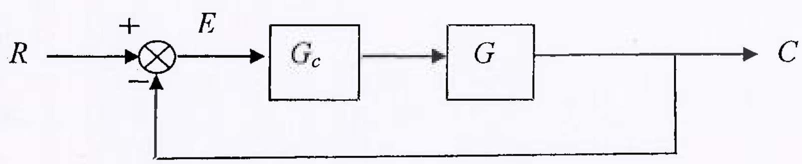

Figure 1

Question 4:

In the system with rate feedback shown in Fig. 2:

(a) Sketch the root locus and find K for a system damping ratio 0.5 for the dominating poles.

(b) Find the steady-state errors for step and ramp inputs for K of part (a).

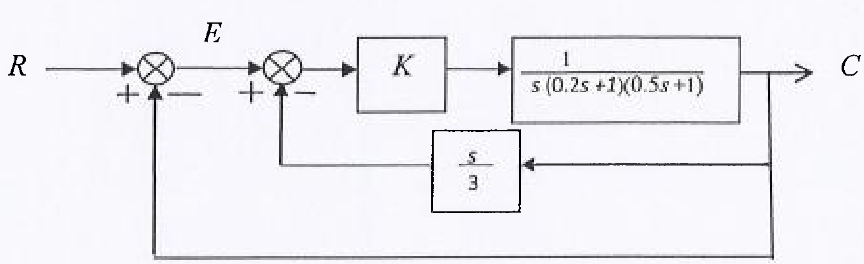

Figure 2

Question 5:

Use the Routh-Hurwitz stability criterion to determine the stability of systems with the following characteristic equations.

(a) $s^4 + 10s^3 + 33s^2 + 46s + 30 = 0$

(b) $s^4 + s^3 + 3s^2 + 2s + 5 = 0$

(c) $s^3 + 2s^3 + 3s + 6 = 0$

Question 6:

In Fig. 3 with $G(s) = 1/[(s + 2)(s + 10)]$:

(a) Calculate the unit step responses for $K = 7$ and $K = 20$.
(b) Verify the steady-state error values of these responses directly.
(c) Compare the responses on the basis of settling time and nature of the response.

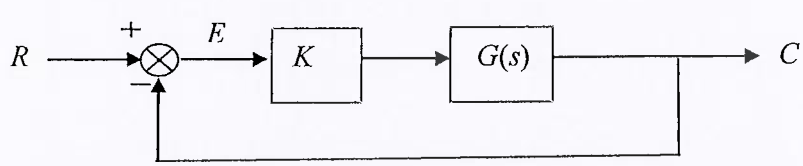

Figure 3

Laplace Transform Table

Laplace transform $F(s)$	Time function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	t^n (n = positive integer)
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t}$ (n = positive integer)
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(e^{-\alpha t}-e^{-\beta t})$ $(\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(\beta e^{-\beta t}-\alpha e^{-\alpha t})$ $(\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1-e^{-\alpha t})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1-e^{-\alpha t}-\alpha t e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\alpha t-1+e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left[t-\frac{1}{\alpha}+\left(t+\frac{2}{\alpha}\right)e^{-\alpha t}\right]$

Laplace Transform Table (continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\dfrac{s}{(s+\alpha)^2}$	$(1-\alpha t)e^{-\alpha t}$
$\dfrac{\omega_n^2}{s^2+\omega_n^2}$	$\sin \omega_n t$
$\dfrac{s}{s^2+\omega_n^2}$	$\cos \omega_n t$
$\dfrac{\omega_n^2}{s(s^2+\omega_n^2)}$	$1-\cos \omega_n t$
$\dfrac{\omega_n^2(s+\alpha)}{s^2+\omega_n^2}$	$\omega_n\sqrt{\alpha^2+\omega_n^2}\sin(\omega_n t+\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\dfrac{\omega_n}{(s+\alpha)(s^2+\omega_n^2)}$	$\dfrac{\omega_n}{\alpha^2+\omega_n^2}e^{-\alpha t}+\dfrac{1}{\sqrt{\alpha^2+\omega_n^2}}\sin(\omega_n t-\theta)$ where $\theta=\tan^{-1}(\omega_n/\alpha)$
$\dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin \omega_n\sqrt{1-\zeta^2}\,t \quad (\zeta<1)$
$\dfrac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$1-\dfrac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\dfrac{s\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\dfrac{-\omega_n^2}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t-\theta)$ where $\theta=\cos^{-1}\zeta \quad (\zeta<1)$
$\dfrac{\omega_n^2(s+\alpha)}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\omega_n\sqrt{\dfrac{\alpha^2-2\alpha\zeta\omega_n+\omega_n^2}{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\tan^{-1}\dfrac{\omega_n\sqrt{1-\zeta^2}}{\alpha-\zeta\omega_n} \quad (\zeta<1)$
$\dfrac{\omega_n^2}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}$	$t-\dfrac{2\zeta}{\omega_n}+\dfrac{1}{\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t+\theta)$ where $\theta=\cos^{-1}(2\zeta^2-1) \quad (\zeta<1)$