National Exams December 2019

16-Elec-A2, Systems & Control

3 hours duration

NOTES:

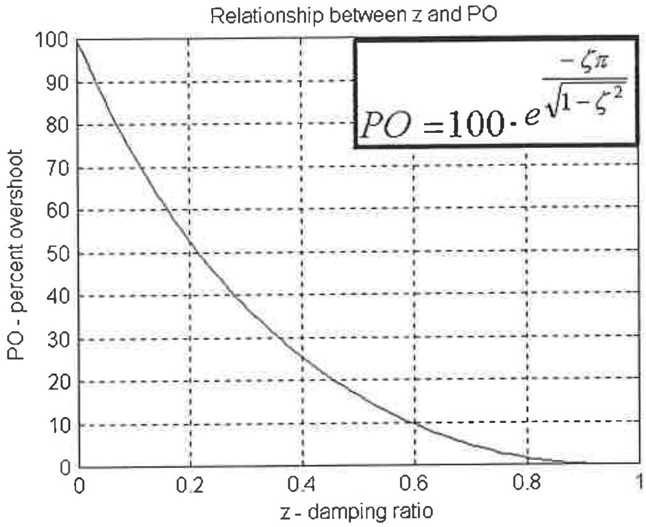
1. This is a CLOSED BOOK EXAM. Only an approved Casio or Sharp calculator is permitted. Candidates are allowed to use a double-sided, **handwritten**, 8.5 by 11" formula and notes sheet. Otherwise, there are no restrictions on the content of the formula sheet. The sheet must be signed and submitted together with the examination paper.
2. If doubt exists as to interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.
3. Five (5) questions constitute a complete paper. **YOU ARE REQUIRED TO COMPLETE QUESTION 1 AND QUESTION 2.** Choose three (3) more questions out of the remaining six. Clearly indicate the questions which should be marked - otherwise, only the first five answers provided will be marked. Each question is of equal value. Weighting of topics within each question is indicated. Partial marks will be given. Clearly show steps taken in answering each question.
4. **Use exam booklets to answer the questions - clearly indicate which question is being answered.**

YOUR MARKS		
QUESTIONS 1 AND 2 ARE COMPULSORY:		
Question 1	20	
Question 2	20	
CHOOSE THREE OUT OF THE REMAINING SIX QUESTIONS:		
Question 3	20	
Question 4	20	
Question 5	20	
Question 6	20	
Question 7	20	
Question 8	20	
TOTAL:	100	

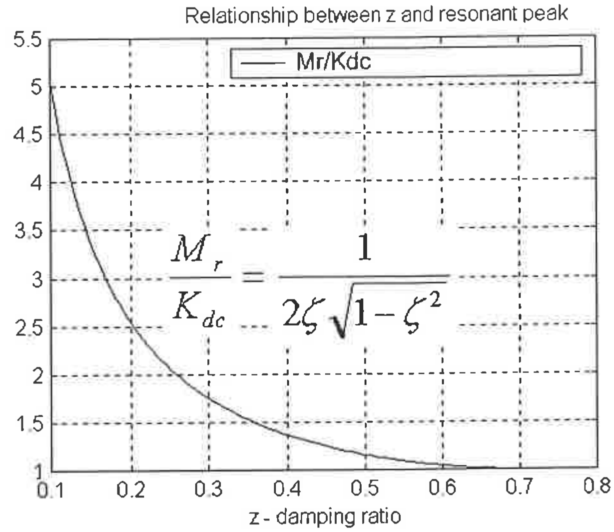
A Short Table of Laplace Transforms

Laplace Transform	Time Function
1	$\sigma(t)$
$\frac{1}{s}$	$1(t)$
$\frac{1}{(s)^2}$	$t \cdot 1(t)$
$\frac{1}{(s)^{k+1}}$	$\frac{t^k}{k!} \cdot 1(t)$
$\frac{1}{s+a}$	$e^{-at} \cdot 1(t)$
$\frac{1}{(s+a)^2}$	$te^{-at} \cdot 1(t)$
$\frac{a}{s(s+a)}$	$(1 - e^{-at}) \cdot 1(t)$
$\frac{a}{s^2+a^2}$	$\sin at \cdot 1(t)$
$\frac{s}{s^2+a^2}$	$\cos at \cdot 1(t)$
$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at} \cdot \cos bt \cdot 1(t)$
$\frac{b}{(s+a)^2+b^2}$	$e^{-at} \cdot \sin bt \cdot 1(t)$
$\frac{a^2+b^2}{s[(s+a)^2+b^2]}$	$\left(1 - e^{-at} \cdot \left(\cos bt + \frac{a}{b} \cdot \sin bt\right)\right) \cdot 1(t)$
$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t\right) \cdot 1(t)$
$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$	$\left(1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cdot \sin\left(\omega_n\sqrt{1-\zeta^2}t + \cos^{-1}\zeta\right)\right) \cdot 1(t)$
$F(s) \cdot e^{-Ts}$	$f(t-T) \cdot 1(t)$
$F(s+a)$	$f(t) \cdot e^{-at} \cdot 1(t)$
$sF(s) - f(0+)$	$\frac{df(t)}{dt}$
$\frac{1}{s}F(s)$	$\int_{0+}^{+\infty} f(t)dt$

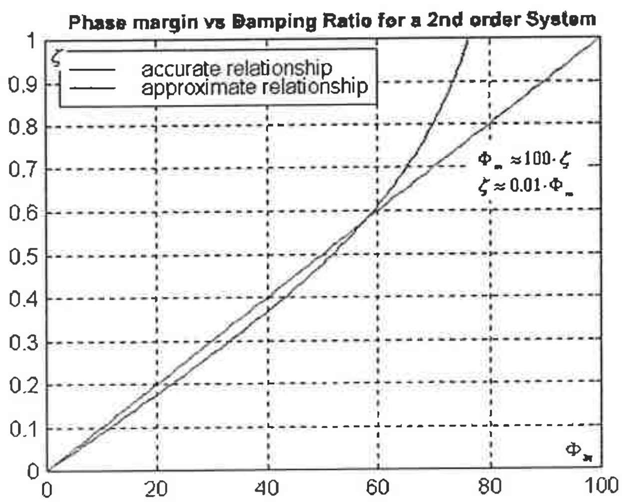
Useful Plots & Formulae



PO vs. Damping Ratio



Resonant Peak vs. Damping Ratio



Second Order Model:

$$G_m(s) = K_{dc} \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ – Damping Ratio (zeta), of the model

ω_n – Frequency of Natural Oscillations of the model

K_{dc} – DC Gain of the model

Definitions for Controllability Matrix, $\mathbf{M_c}$, and Observability Matrix, $\mathbf{M_o}$:

$$\mathbf{M}_c = \begin{bmatrix} \mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} \end{bmatrix} \qquad \mathbf{M}_o = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix}$$

Definition for Transfer Function from State Space:

$$\mathrm{G}(\mathrm{s}) = \mathbf{C} \cdot (\mathrm{s}\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{B} + \mathbf{D}$$

Question 1 (Compulsory)

Controllability and Observability, Transfer Function from State Space Model, Stability, Pole Placement by State Feedback Method, Steady State Error to Step Input.

Consider a linear open loop system described by the following state space model:

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-12 & 1 \\ 1 & 0\end{bmatrix} \cdot \begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}1 \\ 0\end{bmatrix} \cdot u$$

$$y = \begin{bmatrix}1 & 8\end{bmatrix} \cdot \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} + [0] \cdot u$$

1) **(4 marks)** Determine if the open loop system is observable and/or controllable.
2) **(6 marks)** Find the open loop system transfer function, $G_{open}(s) = \frac{Y(s)}{U(s)}$, system poles and zeros. Is the open loop system stable?
3) **(5 marks)** Choose the appropriate locations for the closed loop system poles so that the following specifications will be met:
   - Percent Overshoot, PO, of the compensated closed loop system step response is 0%.
   - The Settling Time, $T_{settle(\pm 2\%)}$, of the compensated closed loop system step response is to be no more than 0.25 seconds.
   - Steady State Error for the unit step input to the compensated closed loop system step response is 0%;
4) **(5 marks)** Place the system in a closed loop configuration with the reference input **r** and assume the controller equation to be in the form:

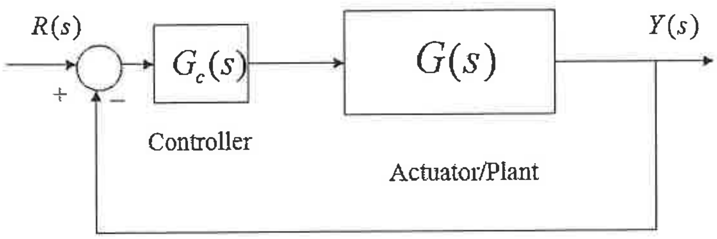
$$u = K \cdot \left(r - \mathbf{k}^T \cdot \mathbf{x}\right)$$

Compute the required values of the state feedback vector gains $\mathbf{k}$ and of the Proportional Gain K.

HINTS: Start with placing one of the closed loop poles so that the pole-zero cancellation in the closed loop occurs. Since the resulting system response will be of a first order model type, choose the second closed loop pole location based on the first order model time constant that will meet the specification. Next, use the Proportional Gain K in the above equation to calibrate for the zero Steady State Error.

Question 2 (Compulsory)

Controller Design in Frequency Domain – Lead Controller, Steady State Error Analysis, Step Response Specifications.



Consider a unit feedback closed loop control system, as shown on the left.

The system is to operate under **Lead Control.** The Lead Controller transfer function is as follows:

$$G_c(s) = K_c \cdot \frac{\tau s + 1}{\alpha \tau s + 1} = \frac{a_1 s + a_0}{b_1 s + 1}$$

Where τ is the so-called Lead Time Constant and $\alpha < 1$.

The process transfer function G(s) is as follows:

$$G(s) = \frac{150(s + 0.6)}{(s + 0.4)(s + 1)^2(s + 15)}$$

Frequency response plots of $G(j\omega)$ are shown in Figure Q2.1. Design requirements for the compensated closed loop system are:

- Steady State Error for the unit step input is to be no more than 5%;
- Percent Overshoot of the compensated closed loop system is to be no more than 15%;
- The Settling Time, $T_{settle(\pm 2\%)}$, is to be no more than 0.5 seconds.
- The Rise Time, $T_{rise(0-100\%)}$, is to be no more than 0.25 seconds.

1) **(5 marks)** Figure Q2.1 shows Phase Margin of the uncompensated system (Φ_{m_u}) and the crossover frequency of the uncompensated system (ω_{cp_u}). Based on these, estimate the **uncompensated** closed loop step response specs: PO, $e_{ss(step\%)}$, $T_{rise(0-100\%)}$ and $T_{settle(\pm 2\%)}$.

2) **(5 marks)** Calculate the Position Constant for the uncompensated system (K_{pos_u}), then the Position Constant for the compensated system (K_{pos_c}) that would meet the design requirements. Decide what value of the Phase Margin for the compensated system (Φ_{m_c}) would meet the design requirements. Decide what value of the crossover frequency for the compensated system (ω_{cp_c}) would meet the design requirements.

3) **(10 marks)** Calculate the appropriate Lead Controller parameters and clearly write the Lead Controller transfer function $G_c(s)$. Superimpose a rough sketch of the compensated open loop frequency response on top of the open loop frequency response in Figure Q2.1.

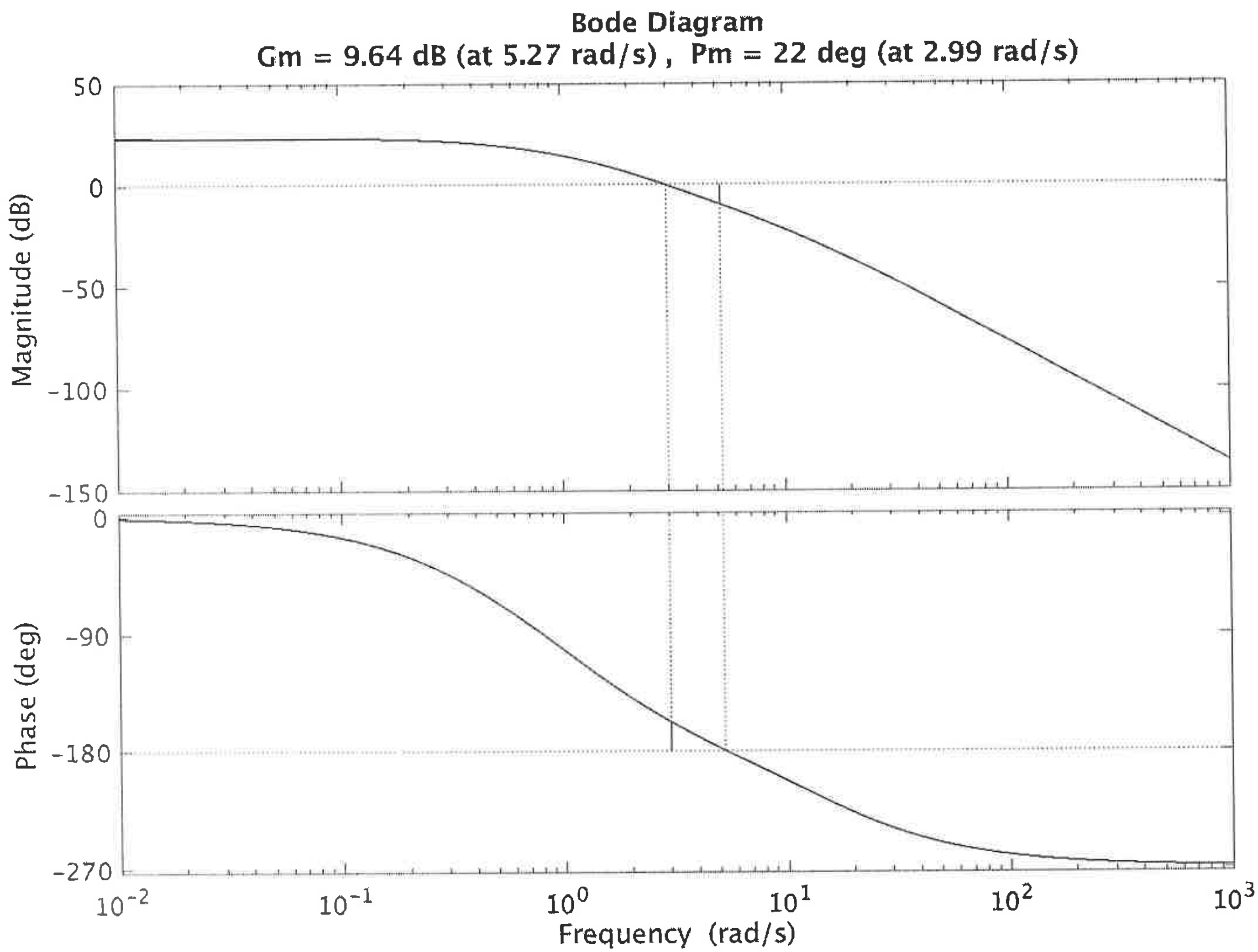
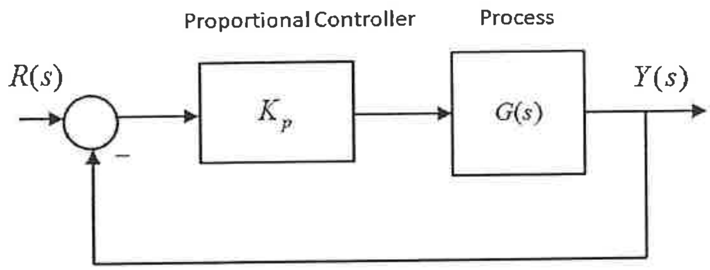


Figure Q2.1 - Frequency Response Plots of the Process $G(j\omega)$ in Question 2

Question 3

Root Locus Analysis and Gain Selection, Stability, Second Order Model, Step Response Specifications.



A unit feedback control system is to be stabilized using a Proportional Controller, as shown on the left. The process transfer function is described as follows:

$$G(s) = \frac{(s+4)(s+8)}{s^2(s+6)}$$

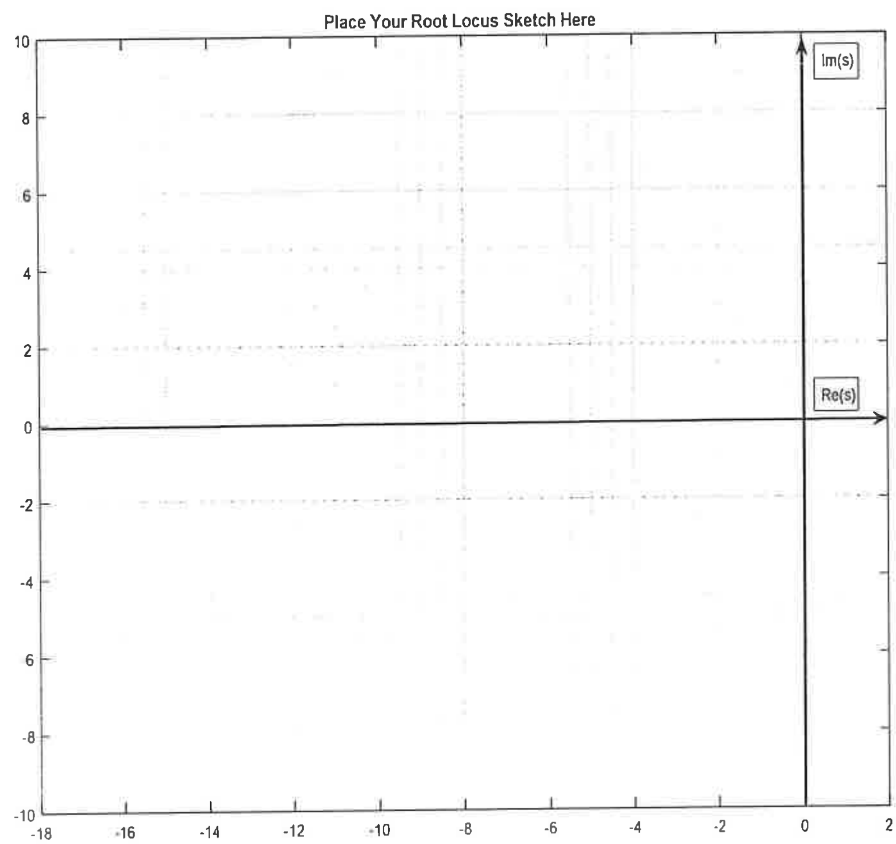


Figure Q3.1 – Space for RL Sketch for Question 3

1) **(10 marks)** Sketch the Root Locus for the system, in the space provided in Figure Q3.1. Calculate all relevant coordinates, such as: asymptotic angles, break-in/away points, the location of the centroid as well as the coordinates of the crossover with the Imaginary axis, i.e. ω_{osc} and the corresponding value of the critical gain, K_{crit}, at which the system becomes marginally stable, if applicable.

2) **(7 marks)** It is required that the unit step response of the Closed Loop system exhibits Percent Overshoot of approximately 5%. Determine the corresponding Proportional Gain value, K_{op}, and calculate estimates of the following specs: Settling Time, $T_{settle(\pm 5\%)}$, Rise Time, $T_{rise(0-100\%)}$, and Steady State Error, $e_{ss(step\%)}$.

3) **(3 marks)** Finally, briefly comment on any possible differences between the expected system response (i.e. of the dominant poles model) and the actual system response.

Question 4

Controller Design by Pole Placement, Response Specifications, Second Order Model.

Consider a closed loop positioning control system working under the Proportional + Integral + Rate Feedback Control, shown in Figure Q4.1. The compensated closed loop step response of this system is to have the following specifications: $PO = 5\%,\ T_{settle(\pm 2\%)} = 0.5$ sec, and $e_{ss(step(\%)} = 0\%$.

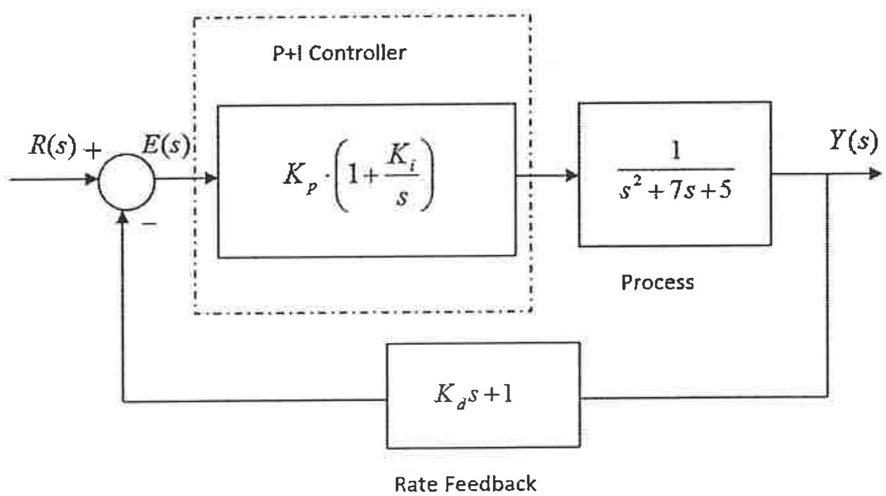


Figure Q4.1 – Closed loop System under PI + Rate Feedback Control

1) **(6 marks)** Derive the closed loop system transfer function in terms of Controller parameters, K_p, K_d and K_i, and write the system Characteristic Equation, $Q(s) = 0$.

2) **(6 marks)** Determine the closed loop system damping ratio, ζ, the frequency of natural oscillations, ω_n, and DC Gain, K_{dc}, to meet the transient and steady state response requirements.

3) **(8 marks)** Choose the pole locations for the closed loop system so that the system two complex conjugate "dominant" poles correspond to the desired second order closed loop model (above) and the third real pole is placed so that **a pole-zero cancellation** in the closed loop transfer function occurs. Compute the required Controller parameters, , K_p, K_d and K_i.

Question 5

System Stability in the s-domain and in the frequency domain: Nyquist Criterion of Stability

Consider a unit feedback loop system under Proportional Control (gain K). The transfer function of its open loop is described as follows:

$$G_{open}(s) = K \cdot \frac{(1+s)}{s(s-0.5)}$$

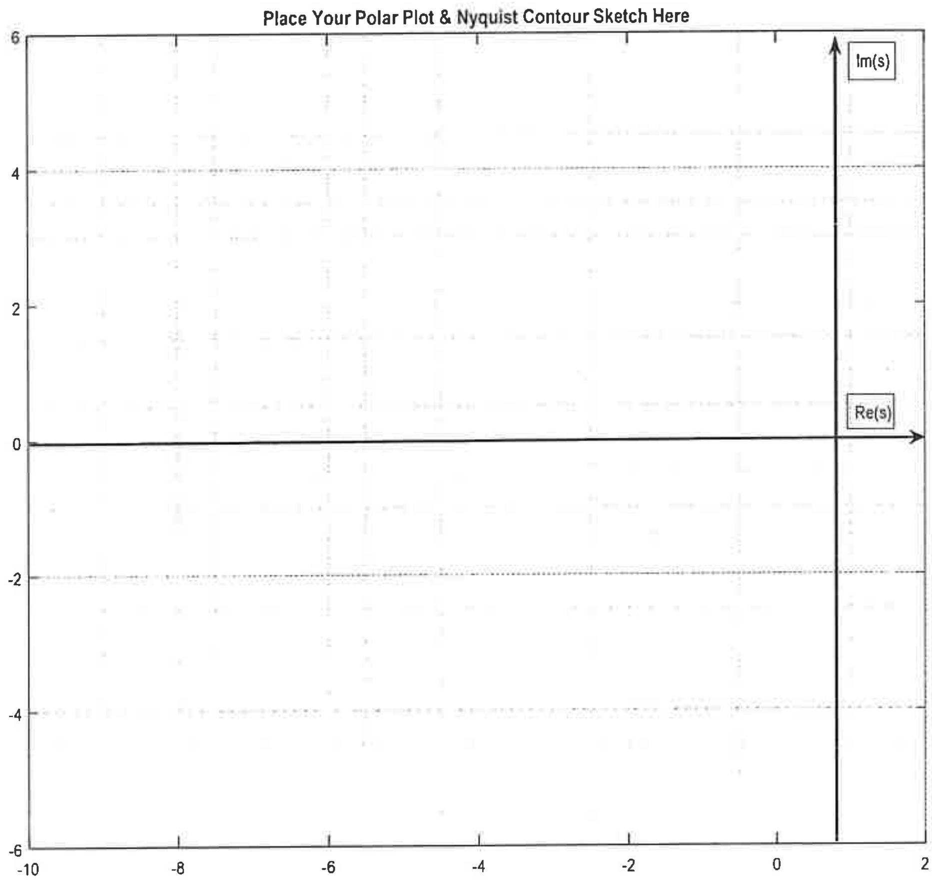


Figure Q5.1 – Space for Polar Plot and Nyquist Contour Sketch for Question 5

1) **(8 marks)** Sketch a polar plot of the normalized open loop transfer function $\frac{G_{open}(j\omega)}{K}$; calculate all relevant coordinates, including the crossovers with the Imaginary and Real axis, and clearly indicate the direction of increasing frequency on the resulting polar plot.
2) **(12 marks)** Apply the Nyquist criterion of stability to establish the range of values of the Proportional Controller gain, K_p, that will result in a stable closed loop system response. NOTE: Assume the clockwise (CW) Γ path in the s-plane, and place the resulting Nyquist contour in Figure Q5.1, on top of the polar plot sketch.

Question 6

Stability in s-domain: Routh Array and Routh-Hurwitz Criterion of Stability.

Consider a certain control system that is to operate under a Proportional + Integral + Derivative (PID) Controller with the transfer function of the controller, $G_{PID}(s)$, as shown below – note that the controller time constants are already substituted ($\tau_i = 0.2$, and $\tau_d = 0.2$). The process is described by the transfer function, $G(s)$, also shown below.

$$G_{PID}(s) = K_p\left(1 + \frac{5}{s} + 0.2s\right) \qquad G(s) = \frac{5}{(s+1)^2}$$

1) **(6 marks)** Assume a unit feedback and derive the closed loop system transfer function $G_{cl}(s)$. Write it in the TF format (polynomial ratio), as a function of the Proportional gain, K_p.

2) (10 marks) Calculate the value(s) of the Critical Gain, K_{crit}, when the system is marginally stable, as well as the frequency of oscillations, ω_{osc}, resulting when $K_p = K_{crit}$.

3) (4 marks) Determine the range of the Proportional Controller gains, K_p, for a safe, stable operation of the closed loop system.

Question 7

State Space System Representations, Analytical Solution for Dynamic System Response.

A certain system is described by a state space model:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-1 & 1\\ -3 & -5\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}-1\\ 0\end{bmatrix}\cdot u$$

$$y = \begin{bmatrix}0 & 1\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\end{bmatrix} + [0]\cdot u$$

1) **(20 marks)** Find the complete solution, $y(t), t \geq 0$, if $u(t) = 1, t \geq 0$ (unit step input), and $x_1(0) = 0, x_2(0) = 0$.

Question 8

Second Order Dominant Poles Model in s-Domain and in Frequency Domain (Open and Closed Loop), Step Response Specifications.

Consider a closed loop system described by the following transfer function $G_{cl}(s)$:

$$G_{cl}(s) = \frac{60(s+50)}{(s+30)(s^2+4s+100)}$$

Your task is to determine an appropriate 2nd order dominant poles model to represent it. The step response of the closed loop transfer function $G_{cl}(s)$ is shown in Figure Q8.1, and the magnitude plot of a frequency response of $G_{cl}(j\omega)$ is shown in Figure Q8.2.

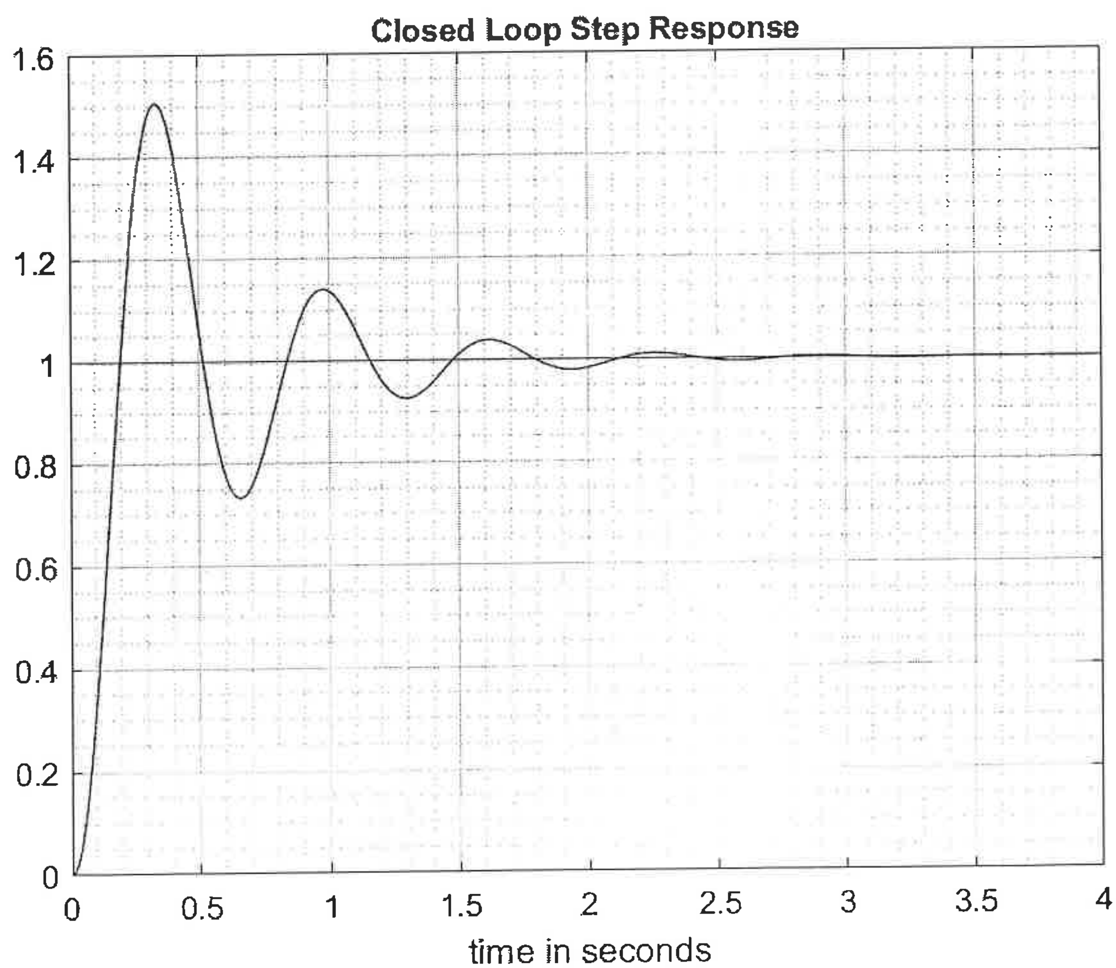


Figure Q8.1: Step Response of the Closed Loop Transfer Function $G_{cl}(s)$ in Question 8.

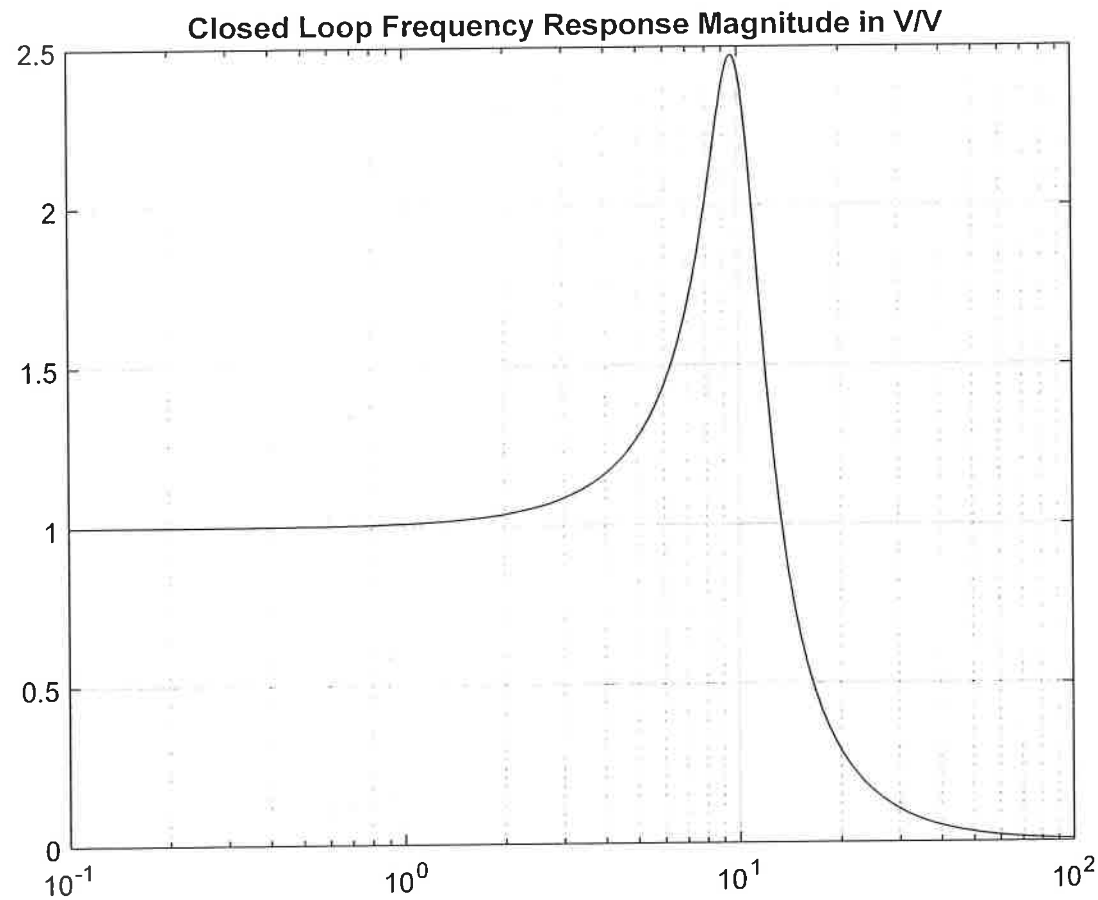


Figure Q8.2 – Closed Loop Frequency Response (Magnitude) Plot of $|G_{cl}(j\omega)|$

1) **(5 marks)** Explain why the second order dominant poles model is appropriate for the closed loop transfer function , $G_{cl}(s)$.. Next, determine the appropriate model for it, and write the transfer function of this model, $G_{m1}(s)$.

2) **(6 marks)** Determine the model parameters from the step response plot in Figure Q8.1, and write the transfer function of this model, $G_{m2}(s)$.

3) **(6 marks)** Determine the model parameters from the plot of $|G_{cl}(j\omega)|$ in Figure Q8.2, and write the transfer function of this model, $G_{m3}(s)$.

4) **(3 marks)** Comment on any discrepancies between the three models, and on which of the three models you would consider the most accurate.