National Exams December 2017

98-Ind-A6, Systems Simulation

3 hours duration

NOTES:

1. If doubt exists as to the interpretation of any question, the candidate is urged to submit with the answer paper, a clear statement of any assumptions made.

2. You are allowed TWO 8.5x11" aid sheets. You may write on both sides of the paper.

3. Candidates may use one of two calculators, the Casio or Sharp approved models.

4. Complete SIX (6) of the EIGHT (8) questions only. If additional questions are answer only the first SIX (6) as they appear in the answer book will be marked.

5. Each question is of equal value and worth 10 points.

6. Before you start, please ensure that the following are attached:
   - Common Discrete Distributions (Source: Table 3-1, Rossetti, 2010)
   - Common Continuous Distributions (Source: Table 3-2, Rossetti, 2010)
   - Student t-distribution table is attached
   - Chi-squared distribution table is attached

1. Short answer questions **(1-3 lines maximum).**

a) Give an example of a system where a finite horizon (terminating) simulation model is appropriate. Explain why.

b) Give one advantage of the Replication/Deletion method over the Batch Means method.

c) Generate two random numbers using the following LCG random number generator:
X_0=27, m=64, c=10, a=10

d) What is the difference between verification and validation?

e) When generating random variants with the Acceptance Rejection Technique, what is the disadvantage of a poor majorizing function?

2. Consider the following data that was collected during different time periods over 5 days.

	Day 1 observations	Day 2 observations	Day 3 observations	Day 4 observations	Day 5 observations
Between 8 and 9	3.3	3.5	3.1	3.7	3.5
Between 9 and 10	3.4	4.1	3.3	3.9	3.4
Between 10 and 11	3.9	3.8	4.1	3.7	3.8

a) Assume the data between 8 and 9 can be modelled with a Poisson distribution with a mean of 3. Generate 2 random variants using random numbers 0.02, 0.30.

b) Test to see if the data is stationary between 9 and 11 (alpha=0.05).

c) Assume we model the three time periods with Poisson distributions and parameters:

	λ
Between 8 and 9	3.4
Between 9 and 10	3.6
Between 10 and 11	3.9

Why is it incorrect to simulate this non-stationary Poisson processes by "changing λ on the fly"?

3. The following data comes from a pilot run of 5 replications of 1 year (with the appropriate warmup period deleted):

Replication:	1	2	3	4	5
Completed Parts:	1377	1350	1299	1324	1578

Calculate the number of replications that will be required for the model to produce an estimate of completed parts to within +/- 50. Let $\alpha=0.05$. Do ***NOT*** approximate using a normal distribution.

4. Paulina Borden, an IE for One Percent Bank, recently completed a simulation of the process used to distribute money to Automated Teller Machines (ATMs). Her simulation modeled the armored vehicles as they left the bank and visited the 20 ATMs in the region. In the original system she modelled the existing behaviour exactly. The existing behaviour sees the drivers use the same route regardless of the time of day. This means the armored vehicles often get stuck in traffic jams. She wants to compare the original system to the following proposed system. In the proposed system the routes used by the armored vehicles will be different at different times of the day and will be chosen in an effort to avoid traffic jams.

a) Should a paired t-test be used to compare the results from the original system to the proposed system? Explain why or why not.

b) Use the following data and determine if the proposed system should be adopted or not (assume 90% confidence and two independent samples with equal variance). The data represents the minutes needed to complete a single route.

Replication	Original	Proposed
1	177	175
2	186	172
3	188	172
4	172	186
5	189	188
6	184	170
7	173	166
8	180	186
9	172	177
10	182	170

5. Robert Border, an IE for South Shore Machines, is developing a simulation model of a production process. One of the issues in the plant that has come up is the absenteeism and the availability of staff. Accordingly, Robert has collected statistics on the number of unplanned absences over the past 100 days. Robert has compiled the data into the following table:

Number of Employees Absent:	0	1	2	3	4	5	6	7	8	9	10
Frequency:	0	3	5	9	21	22	24	11	4	1	0

Given this sample:

a) Calculate the average number of employees absent on any given day.

b) Calculate the variance for the number of employees absent on any given day.

c) Hypothesize an appropriate distribution for this data sample and give a brief rationale for selecting that particular distribution.

6. 2. Consider an M/M/1 system with two customer types: high priority customers and low priority customers. Low priority customers are served first-come-first-serve unless a high priority customer is waiting, in which case, the high priority customer is served next.

a) Complete the following table:

Customer	Priority	Interarrival time	Arrival Time	Service Time	Start Service	End Service
1	High		0	0.7		
2	Low	0.3		1		
3	Low	0.5		0.8		
4	High	0.3		1		
5	Low	1.2		0.5		

b) What is the average waiting time for high priority customers?

c) Assuming both customer types wait in the same queue, what is the average queue length?

7. When solving problems with simulation there are a number of steps which should always be completed. Consider the following problem and briefly explain 8 steps necessary to solve it:

The ambulance service in Nova Scotia has multiple stations where ambulances wait for calls and where supplies, such as medication, bandages, disinfectants, etc., are stored. The ambulance service feels the supplies are not well managed and wants a simulation study completed.

8. Consider the check-in process at an airport that opens 4 hours before the flight. Passengers are either business class or economy class. Business class passengers arrive at rate of 18 / hour (Poisson distributed) and economy class passengers arrive at a rate of 75 / hour (Poisson distributed). The total number of business passengers is 50 and the total number of economy passengers is 200.

The check in process consists of two main steps. First, passengers must check-in for the flights (unless they have already checked-in online) and second, they must drop off their luggage if they have any. Afterwards, they proceed to security. You don't need to model security.

The characteristics of each passenger type are below:

Passenger Type	Checked-in Online	No Luggage	1 Piece of luggage	2 Pieces of luggage
Economy	50%	20%	50%	30%
Business	75%	50%	45%	5%

Passengers who have checked-in online and who do not have luggage go straight to security and exit the system. Note that I still want you to model these passengers.

Passengers who have not checked-in online must go to an automated kiosk machine to check-in. There are 50 kiosk machines. Business class customers can use any of the 50 kiosks. Economy class customers cannot use the 15 kiosks which are reserved for business class customers only. It takes both economy and business passengers Unif(3,7) minutes to check in.

Once checked-in, those without luggage proceed to security and exit the system.

Passengers with luggage must go the luggage drop off area. There are two queues, one for economy passengers and one for business passengers. Agents 1, 2, and 3 are dedicated solely to economy customers and Agent 4 is dedicated solely to business customers. Agent 5 will serve either queue but gives priority to business class customers. The service time for the agents are:

	Service Time Business Class (Minutes)	Service Time Economy Class (Minutes)
Agents 1,2,3	Not Possible	Expo(9)
Agent 4	Expo(5)	Not Possible
Agent 5	Unif(2,5)	Unif(3,9)

If an economy class passenger has one piece of luggage and finds more than 15 people in the economy queue, they will elect to carry their luggage onto the plane instead of waiting in line. In other words, they bulk and proceed straight to security.

Business class passengers who find more than 3 people in their luggage queue get very upset. When it is their turn to drop off their baggage, 20% of them demand to speak to a supervisor (the other 80% of upset passengers do not). The agent waits with the passenger until the supervisor is free (there is only 1 supervisor). The supervisor and the agent discuss the passenger's concern for 3 minutes. Afterwards, the agent and the supervisor spend another Unif(2,4) minutes to process the passenger's luggage before the passenger goes to security.

After a passenger's luggage is processed they go to security and exit the system.

The following should be computed by the model

1. On average, how long does it take before all 250 passengers exit the system?
2. On average, how many people proceed to security with less than 1 hour remaining before their flight?
3. Passengers who are not at security 15 minutes before their flight will miss their flight. What is the probability that at least 1 person will miss their flight?

Sketch a process flow diagram of a simulation for this system. The diagram does not need to be pseudo-code for your simulation, but there should be a correlation between your flow diagram and how the model could be coded.

nmon Discrete Distributions

Distribution, Random Variable X	PMF	E[X] & V[X]
Bernoulli(p) The number of successes in one trial	$P(X=1)=p$ $P(X=0)=1-p$	$E[X]=p$ $V[X]=p(1-p)$
Binomial(n,p) The number of successes in n Bernoulli trials	$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$ $0\le p\le 1 \quad x=0,1,\ldots,n$	$E[X]=np$ $V[X]=np(1-p)$
Geometric(p) The number of trials until the first success in a sequence of Bernoulli trials	$P(X=x)=p(1-p)^{x-1}$ $0\le p\le 1 \quad x=1,2,\ldots$	$E[X]=1/p$ $V[X]=(1-p)/p^2$
Negative Binomial(r,p), Defn. 1 The number of trials until the r^{th} success in a sequence of Bernoulli trials	$P(X=x)=\binom{x-1}{r-1}p^r(1-p)^{x-r}$ $0\le p\le 1 \quad x=r,r+1,\ldots$	$E[X]=r/p$ $V[X]=r(1-p)/p^2$
Negative Binomial, Defn. 2 The number of failures prior to the r^{th} success in a sequence of Bernoulli trials	$P(Y=y)=\binom{y+r-1}{r-1}p^r(1-p)^y$ $0\le p\le 1 \quad y=0,1,\ldots$	$E[Y]=r(1-p)/p$ $V[Y]=r(1-p)/p^2$
Poisson(λ) The number of event occurrences during a specified period of time	$P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!} \quad x=0,1,\ldots$ $\lambda>0$	$E[X]=\lambda$ $V[X]=\lambda$
Discrete Uniform(a,b)	$P(X=x)=\frac{1}{b-a+1}$ $a\le b \quad x=a,a+1,\ldots,b$	$E[X]=(b+a)/2$ $V[X]=\left((b-a+1)^2-1\right)/12$
Discrete Uniform	$P(X=x)=\frac{1}{n}$ $x=x_1,x_2,x_3,\ldots,x_n; n\in\mathbb{N}$	$E[X]=\frac{1}{n}\sum_{i=1}^{n}x_i$ $V[X]=\frac{1}{n}\sum_{i=1}^{n}x_i^2-(E[X])^2$

Note: $\binom{n}{x}=\frac{n!}{(n-x)!x!}$

ion Continuous Distributions

Distribution	$F(x)$	$E[X]$	$V[X]$
Uniform(a,b)	$\frac{x-a}{b-a} \quad a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Normal(μ,σ^2)	No Closed Form	μ	σ^2
Exponential(θ)	$1-e^{-x/\theta}$	θ	θ^2
Erlang(θ, r)	$1-\sum_{n=0}^{r-1} \frac{e^{-x/\theta}(x/\theta)^n}{n!}$	$r\theta$	$r\theta^2$
Gamma(β, α)	If r is a positive integer, see Erlang; otherwise, no closed form	$\alpha\beta$	$\alpha\beta^2$
Weibull(β, α) beta = *scale* alpha = *shape*	$1-e^{-(x/\beta)^\alpha}$	$\frac{\beta}{\alpha}\Gamma\left(\frac{1}{\alpha}\right)$	$\frac{\beta^2}{\alpha}\left\{2\Gamma\left(\frac{2}{\alpha}\right)-\frac{1}{\alpha}\left(\Gamma\left(\frac{1}{\alpha}\right)\right)^2\right\}$
Lognormal (μ_l, σ_l^2)	No Closed Form $\mu = \ln\left(\mu_l/\sqrt{\sigma_l^2+\mu_l^2}\right)$ $\sigma^2 = \ln\left((\sigma_l^2+\mu_l^2)/\mu_l^2\right)$	$e^{\mu+\frac{\sigma^2}{2}}$	$e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$
Beta(α_1,α_2)	No Closed Form	$\frac{\alpha_1}{\alpha_1+\alpha_2}$	$\frac{\alpha_1\alpha_2}{(\alpha_1+\alpha_2)^2(\alpha_1+\alpha_2+1)}$
Triangular(a, m, b) a = minimum b = maximum m = mode	$\begin{cases} \frac{(x-a)^2}{(b-a)(m-a)} & a \le x \le m \\ 1-\frac{(b-x)^2}{(b-a)(b-m)} & m \le x \le b \end{cases}$	$\frac{a+b+m}{3}$	$\frac{a^2+b^2+m^2-ab-am-bm}{18}$

A.4 Percentage Points $t_{\alpha,\nu}$ of the Student t-Distribution (ν degrees of freedom)

ν \ α	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
31	1.309	1.696	2.040	2.453	2.744	3.022	3.375	3.633
32	1.309	1.694	2.037	2.449	2.738	3.015	3.365	3.622
33	1.308	1.692	2.035	2.445	2.733	3.008	3.356	3.611
34	1.307	1.691	2.032	2.441	2.728	3.002	3.348	3.601
35	1.306	1.690	2.030	2.438	2.724	2.996	3.340	3.591
40	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
45	1.301	1.679	2.014	2.412	2.690	2.952	3.281	3.520
50	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
∞	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.01
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38